CHAPTER 12

UNSYMMETRICAL FAULTS

Most of the faults that occur on power systems are unsymmetrical faults, which may consist of unsymmetrical short circuits, unsymmetrical faults through impedances, or open conductors. Unsymmetrical faults occur as single line-to-ground faults, line-to-line faults, or double line-to-ground faults. The path of the fault current from line to line or line to ground may or may not contain impedance. One or two open conductors result in unsymmetrical faults, through either the breaking of one or two conductors or the action of fuses and other devices that may not open the three phases simultaneously. Since any unsymmetrical fault causes unbalanced currents to flow in the system, the method of symmetrical components is very useful in an analysis to determine the currents and voltages in all parts of the system after the occurrence of the fault. We consider faults on a power system by applying Thévenin's theorem, which allows us to find the current in the fault by replacing the entire system by a single generator and series impedance, and we show how the bus impedance matrix is applied to the analysis of unsymmetrical faults.

12.1 UNSYMMETRICAL FAULTS ON POWER SYSTEMS

In the derivation of equations for the symmetrical components of currents and voltages in a general network the currents flowing *out* of the original balanced

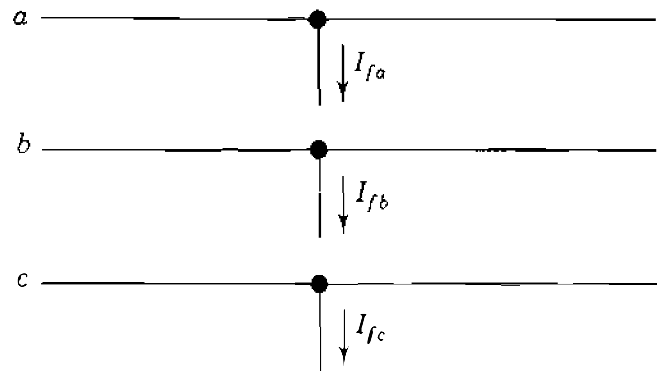


FIGURE 12.1
Three conductors of a three-phase system. The stubs carrying currents I_{fa}, I_{fb}, and I_{fc} may be interconnected to represent different types of faults.

system from phases a, b, and c at the fault point will be designated as I_{fa}, I_{fb}, and I_{fc}, respectively. We can visualize these currents by referring to Fig. 12.1, which shows the three lines a, b, and c of the three-phase system at the part of the network where the fault occurs. The flow of current from each line into the fault is indicated by arrows shown on the diagram beside *hypothetical stubs* connected to each line at the fault location. Appropriate connections of the stubs represent the various types of fault. For instance, direct connection of stubs b and c produces a line-to-line fault through zero impedance. The current in stub a is then zero, and I_{fb} equals $-I_{fc}$.

The line-to-ground voltages at *any* bus j of the system *during* the fault will be designated V_{ja}, V_{jb}, and V_{jc}; and we shall continue to use superscripts 1, 2, and 0, respectively, to denote positive-, negative-, and zero-sequence quantities. Thus, for example, $V_{ja}^{(1)}$, $V_{ja}^{(2)}$, and $V_{ja}^{(0)}$ will denote, respectively, the positive-, negative-, and zero-sequence components of the line-to-ground voltage V_{ja} at bus j *during* the fault. The line-to-neutral voltage of phase a at the fault point *before* the fault occurs will be designated simply by V_f, which is a *positive-sequence* voltage since the system is balanced. We met the prefault voltage V_f previously in Sec. 10.3 when calculating the currents in a power system with a symmetrical three-phase fault applied.

A single-line diagram of a power system containing two synchronous machines is shown in Fig. 12.2. Such a system is sufficiently general for equations derived therefrom to be applicable to any balanced system regardless of the complexity. Figure 12.2 also shows the sequence networks of the system. The point where a fault is assumed to occur is marked P, and in this particular example it is called bus k on the single-line diagram and in the sequence networks. Machines are represented by their subtransient internal voltages in series with their subtransient reactances when subtransient fault conditions are being studied.

In Sec. 10.3 we used the bus impedance matrix composed of positive-sequence impedances to determine currents and voltages upon the occurrence of a symmetrical three-phase fault. The method can be easily extended to apply to *unsymmetrical* faults by realizing that the negative- and zero-sequence networks also can be represented by bus impedance matrices. The bus impedance matrix will now be written symbolically for the positive-sequence network in the following form:

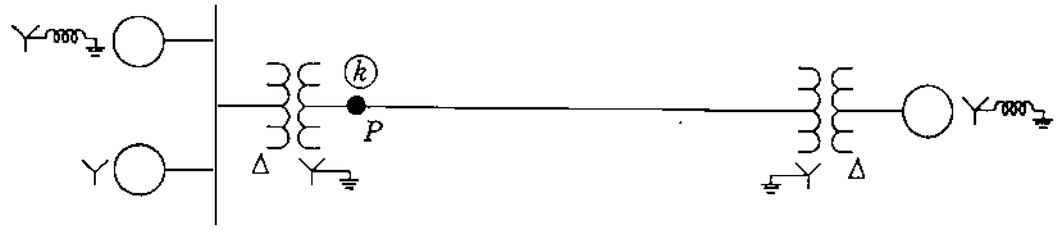
$$\mathbf{Z}_{\text{bus}}^{(1)} = \begin{array}{c} \\ ① \\ ② \\ \\ ⓚ \\ \\ Ⓝ \end{array} \begin{array}{c} \begin{array}{cccccc} ① & ② & & ⓚ & & Ⓝ \end{array} \\ \begin{bmatrix} Z_{11}^{(1)} & Z_{12}^{(1)} & \cdots & Z_{1k}^{(1)} & \cdots & Z_{1N}^{(1)} \\ Z_{21}^{(1)} & Z_{22}^{(1)} & \cdots & Z_{2k}^{(1)} & \cdots & Z_{2N}^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1}^{(1)} & Z_{k2}^{(1)} & \cdots & Z_{kk}^{(1)} & \cdots & Z_{kN}^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1}^{(1)} & Z_{N2}^{(1)} & \cdots & Z_{Nk}^{(1)} & \cdots & Z_{NN}^{(1)} \end{bmatrix} \end{array} \qquad (12.1)$$

Similarly, the bus impedance matrices for the negative- and zero-sequence networks will be written

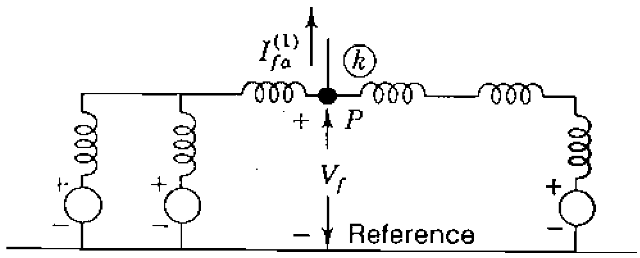
$$\mathbf{Z}_{\text{bus}}^{(2)} = \begin{array}{c} \\ ① \\ ② \\ \\ ⓚ \\ \\ Ⓝ \end{array} \begin{array}{c} \begin{array}{cccccc} ① & ② & & ⓚ & & Ⓝ \end{array} \\ \begin{bmatrix} Z_{11}^{(2)} & Z_{12}^{(2)} & \cdots & Z_{1k}^{(2)} & \cdots & Z_{1N}^{(2)} \\ Z_{21}^{(2)} & Z_{22}^{(2)} & \cdots & Z_{2k}^{(2)} & \cdots & Z_{2N}^{(2)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1}^{(2)} & Z_{k2}^{(2)} & \cdots & Z_{kk}^{(2)} & \cdots & Z_{kN}^{(2)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1}^{(2)} & Z_{N2}^{(2)} & \cdots & Z_{Nk}^{(2)} & \cdots & Z_{NN}^{(2)} \end{bmatrix} \end{array} \qquad (12.2)$$

$$\mathbf{Z}_{\text{bus}}^{(0)} = \begin{array}{c} \\ ① \\ ② \\ \\ ⓚ \\ \\ Ⓝ \end{array} \begin{array}{c} \begin{array}{cccccc} ① & ② & & ⓚ & & Ⓝ \end{array} \\ \begin{bmatrix} Z_{11}^{(0)} & Z_{12}^{(0)} & \cdots & Z_{1k}^{(0)} & \cdots & Z_{1N}^{(0)} \\ Z_{21}^{(0)} & Z_{22}^{(0)} & \cdots & Z_{2k}^{(0)} & \cdots & Z_{2N}^{(0)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1}^{(0)} & Z_{k2}^{(0)} & \cdots & Z_{kk}^{(0)} & \cdots & Z_{kN}^{(0)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1}^{(0)} & Z_{N2}^{(0)} & \cdots & Z_{Nk}^{(0)} & \cdots & Z_{NN}^{(0)} \end{bmatrix} \end{array}$$

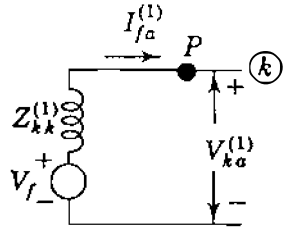
Thus, $Z_{ij}^{(1)}$, $Z_{ij}^{(2)}$, and $Z_{ij}^{(0)}$ denote representative elements of the bus impedance matrices for the positive-, negative-, and zero-sequence networks, respectively. If so desired, each of the networks can be replaced by its Thévenin equivalent between any one of the buses and the reference node.



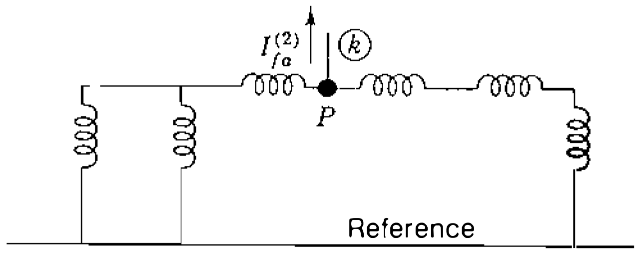
(a) Single-line diagram of balanced three-phase system



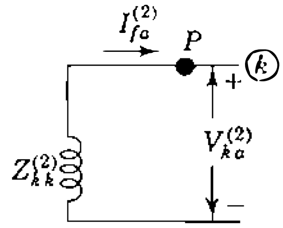
(b) Positive-sequence network



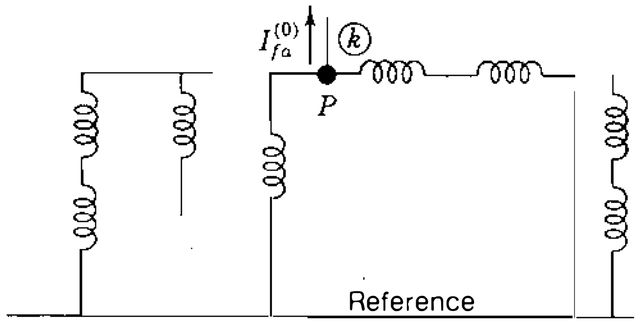
(e) Thévenin equivalent of the positive-sequence network



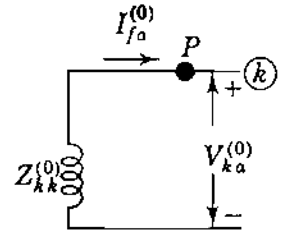
(c) Negative-sequence network



(f) Thévenin equivalent of the negative-sequence network



(d) Zero-sequence network



(g) Thévenin equivalent of the zero-sequence network

FIGURE 12.2
Single-line diagram of a three-phase system, the three sequence networks of the system, and the Thévenin equivalent of each network for a fault at P, which is called bus (k).

The Thévenin equivalent circuit between the fault point P and the reference node in each sequence network is shown adjacent to the diagram of the corresponding network in Fig. 12.2. As in Chap. 10, the voltage source in the positive-sequence network and its Thévenin equivalent circuit is V_f, the prefault voltage to neutral at the fault point P, which happens to be bus (k) in this illustration. The Thévenin impedance measured between point P and the reference node of the positive-sequence network is $Z_{kk}^{(1)}$, and its value depends on the values of the reactances used in the network. We recall from Chap. 10 that subtransient reactances of generators and 1.5 times the subtransient reactances (or else the transient reactances) of synchronous motors are the values used in calculating the symmetrical current to be interrupted.

There are no negative- or zero-sequence currents flowing before the fault occurs, and the prefault voltages are zero at all buses of the negative- and zero-sequence networks. Therefore, the prefault voltage between point P and the reference node is zero in the negative- and zero-sequence networks and no electromotive forces (emfs) appear in their Thévenin equivalents. The negative- and zero-sequence impedances between point P at bus ⓚ and the reference node in the respective networks are represented by the Thévenin impedances $Z_{kk}^{(2)}$ and $Z_{kk}^{(0)}$—diagonal elements of $\mathbf{Z}_{\text{bus}}^{(2)}$ and $\mathbf{Z}_{\text{bus}}^{(0)}$, respectively.

Since I_{fa} is the current flowing *from* the system *into* the fault, its symmetrical components $I_{fa}^{(1)}$, $I_{fa}^{(2)}$, and $I_{fa}^{(0)}$ flow *out* of the respective sequence networks and their equivalent circuits at point P, as shown in Fig. 12.2. Thus, the currents $-I_{fa}^{(1)}$, $-I_{fa}^{(2)}$, and $-I_{fa}^{(0)}$ represent *injected currents into* the faulted bus ⓚ of the positive-, negative-, and zero-sequence networks due to the fault. These current injections cause voltage *changes* at the buses of the positive-, negative-, and zero-sequence networks, which can be calculated from the bus impedance matrices in the manner demonstrated in Sec. 10.3. For instance, due to the injection $-I_{fa}^{(1)}$ into bus ⓚ, the voltage changes in the positive-sequence network of the N-bus system are given in general terms by

$$\begin{bmatrix} \Delta V_{1a}^{(1)} \\ \Delta V_{2a}^{(1)} \\ \vdots \\ \Delta V_{ka}^{(1)} \\ \vdots \\ \Delta V_{Na}^{(1)} \end{bmatrix} = \begin{array}{c} \\ (1) \\ (2) \\ \\ (k) \\ \\ (N) \end{array} \begin{array}{c} \begin{array}{ccccccc} (1) & (2) & & (k) & & (N) \end{array} \\ \begin{bmatrix} Z_{11}^{(1)} & Z_{12}^{(1)} & \cdots & Z_{1k}^{(1)} & \cdots & Z_{1N}^{(1)} \\ Z_{21}^{(1)} & Z_{22}^{(1)} & \cdots & Z_{2k}^{(1)} & \cdots & Z_{2N}^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1}^{(1)} & Z_{k2}^{(1)} & \cdots & Z_{kk}^{(1)} & \cdots & Z_{kN}^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1}^{(1)} & Z_{N2}^{(1)} & \cdots & Z_{Nk}^{(1)} & \cdots & Z_{NN}^{(1)} \end{bmatrix} \end{array} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_{fa}^{(1)} \\ \vdots \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -Z_{1k}^{(1)} I_{fa}^{(1)} \\ -Z_{2k}^{(1)} I_{fa}^{(1)} \\ \vdots \\ -Z_{kk}^{(1)} I_{fa}^{(1)} \\ \vdots \\ -Z_{Nk}^{(1)} I_{fa}^{(1)} \end{bmatrix} \tag{12.3}$$

This equation is quite similar to Eq. (10.15) for symmetrical faults. Note that only column k of $\mathbf{Z}_{\text{bus}}^{(1)}$ enters into the calculations. In industry practice, it is customary to regard all prefault currents as being zero and to designate the voltage V_f as the positive-sequence voltage at all buses of the system before the fault occurs. Superimposing the changes of Eq. (12.3) on the prefault voltages then yields the total positive-sequence voltage of phase a at each bus during the fault,

$$\begin{bmatrix} V_{1a}^{(1)} \\ V_{2a}^{(1)} \\ \vdots \\ V_{ka}^{(1)} \\ \vdots \\ V_{Na}^{(1)} \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ \vdots \\ V_f \\ \vdots \\ V_f \end{bmatrix} + \begin{bmatrix} \Delta V_{1a}^{(1)} \\ \Delta V_{2a}^{(1)} \\ \vdots \\ \Delta V_{ka}^{(1)} \\ \vdots \\ \Delta V_{Na}^{(1)} \end{bmatrix} = \begin{bmatrix} V_f - Z_{1k}^{(1)} I_{fa}^{(1)} \\ V_f - Z_{2k}^{(1)} I_{fa}^{(1)} \\ \vdots \\ V_f - Z_{kk}^{(1)} I_{fa}^{(1)} \\ \vdots \\ V_f - Z_{Nk}^{(1)} I_{fa}^{(1)} \end{bmatrix} \tag{12.4}$$

This equation is similar to Eq. (10.18) for symmetrical faults, the only difference being the added superscripts and subscripts denoting the positive-sequence components of the phase a quantities.

Equations for the negative- and zero-sequence voltage changes due to the fault at bus (k) of the N-bus system are similarly written with the superscripts in Eq. (12.3) changed from 1 to 2 and from 1 to 0, respectively. Because the prefault voltages are zero in the negative- and zero-sequence networks, the voltage changes express the *total* negative- and zero-sequence voltages during the fault, and so we have

$$\begin{bmatrix} V_{1a}^{(2)} \\ V_{2a}^{(2)} \\ \vdots \\ V_{ka}^{(2)} \\ \vdots \\ V_{Na}^{(2)} \end{bmatrix} = \begin{bmatrix} -Z_{1k}^{(2)} I_{fa}^{(2)} \\ -Z_{2k}^{(2)} I_{fa}^{(2)} \\ \vdots \\ -Z_{kk}^{(2)} I_{fa}^{(2)} \\ \vdots \\ -Z_{Nk}^{(2)} I_{fa}^{(2)} \end{bmatrix} \qquad \begin{bmatrix} V_{1a}^{(0)} \\ V_{2a}^{(0)} \\ \vdots \\ V_{ka}^{(0)} \\ \vdots \\ V_{Na}^{(0)} \end{bmatrix} = \begin{bmatrix} -Z_{1k}^{(0)} I_{fa}^{(0)} \\ -Z_{2k}^{(0)} I_{fa}^{(0)} \\ \vdots \\ -Z_{kk}^{(0)} I_{fa}^{(0)} \\ \vdots \\ -Z_{Nk}^{(0)} I_{fa}^{(0)} \end{bmatrix} \tag{12.5}$$

When the fault is at bus (k), note that only the entries in columns k of $\mathbf{Z}_{\text{bus}}^{(2)}$ and $\mathbf{Z}_{\text{bus}}^{(0)}$ are involved in the calculations of negative- and zero-sequence voltages. Thus, knowing the symmetrical components $I_{fa}^{(1)}$, $I_{fa}^{(1)}$, and $I_{fa}^{(2)}$ of the fault currents at bus (k), we can determine the sequence voltages at *any* bus (j) of the system from the jth rows of Eqs. (12.4) and (12.5). That is, during the fault at bus $\textcircled{k}$ the voltages at any bus $\textcircled{j}$ are

$$\begin{aligned} V_{ja}^{(0)} &= \quad\; - Z_{jk}^{(0)} I_{fa}^{(0)} \\ V_{ja}^{(1)} &= V_f - Z_{jk}^{(1)} I_{fa}^{(1)} \\ V_{ja}^{(2)} &= \quad\; - Z_{jk}^{(2)} I_{fa}^{(2)} \end{aligned} \tag{12.6}$$

If the prefault voltage at bus $\textcircled{j}$ is not V_f, then we simply replace V_f in Eq. (12.6) by the actual value of the prefault (positive-sequence) voltage at that bus. Since V_f is *by definition* the actual prefault voltage at the faulted bus $\textcircled{k}$, we always have at that bus

$$\begin{aligned} V_{ka}^{(0)} &= \quad\; - Z_{kk}^{(0)} I_{fa}^{(0)} \\ V_{ka}^{(1)} &= V_f - Z_{kk}^{(1)} I_{fa}^{(1)} \\ V_{ka}^{(2)} &= \quad\; - Z_{kk}^{(2)} I_{fa}^{(2)} \end{aligned} \tag{12.7}$$

and these are the terminal voltage equations for the Thévenin equivalents of the sequence networks shown in Fig. 12.2.

It is important to remember that the currents $I_{fa}^{(0)}$, $I_{fa}^{(1)}$, and $I_{fa}^{(2)}$ are symmetrical-component currents in the stubs hypothetically attached to the system at the fault point. These currents take on values determined by the particular type of fault being studied, and once they have been calculated, they can be regarded as negative injections into the corresponding sequence networks. If the system has Δ-Y transformers, some of the sequence voltages calculated from Eqs. (12.6) may have to be shifted in phase angle before being combined with other components to form the new bus voltages of the faulted system. There are no phase shifts involved in Eq. (12.7) when the voltage V_f at the fault point is chosen as reference, which is customary.

In a system with Δ-Y transformers the open circuits encountered in the zero-sequence network require careful consideration in computer applications of the $\mathbf{Z}_{\text{bus}}$ building algorithm. Consider, for instance, the solidly grounded Y-Δ transformer connected between buses $\textcircled{m}$ and $\textcircled{n}$ of Fig. 12.3(a). The positive- and zero-sequence circuits are shown in Figs. 12.3(b) and 12.3(c), respectively. The negative-sequence circuit is the same as the positive-sequence circuit. It is straightforward to include these sequence circuits in the bus impedance matrices $\mathbf{Z}_{\text{bus}}^{(0)}$, $\mathbf{Z}_{\text{bus}}^{(1)}$, and $\mathbf{Z}_{\text{bus}}^{(2)}$ using the *pictorial* representations shown in the figures. This will be done in the sections which follow when Y-Δ transformers are present. Suppose, however, that we wish to represent *removal* of the transformer connections from bus $\textcircled{n}$ in a computer algorithm which cannot avail of pictorial representations. We can easily undo the connections to bus $\textcircled{n}$ in the positive- and negative-sequence networks by applying the building algorithm to

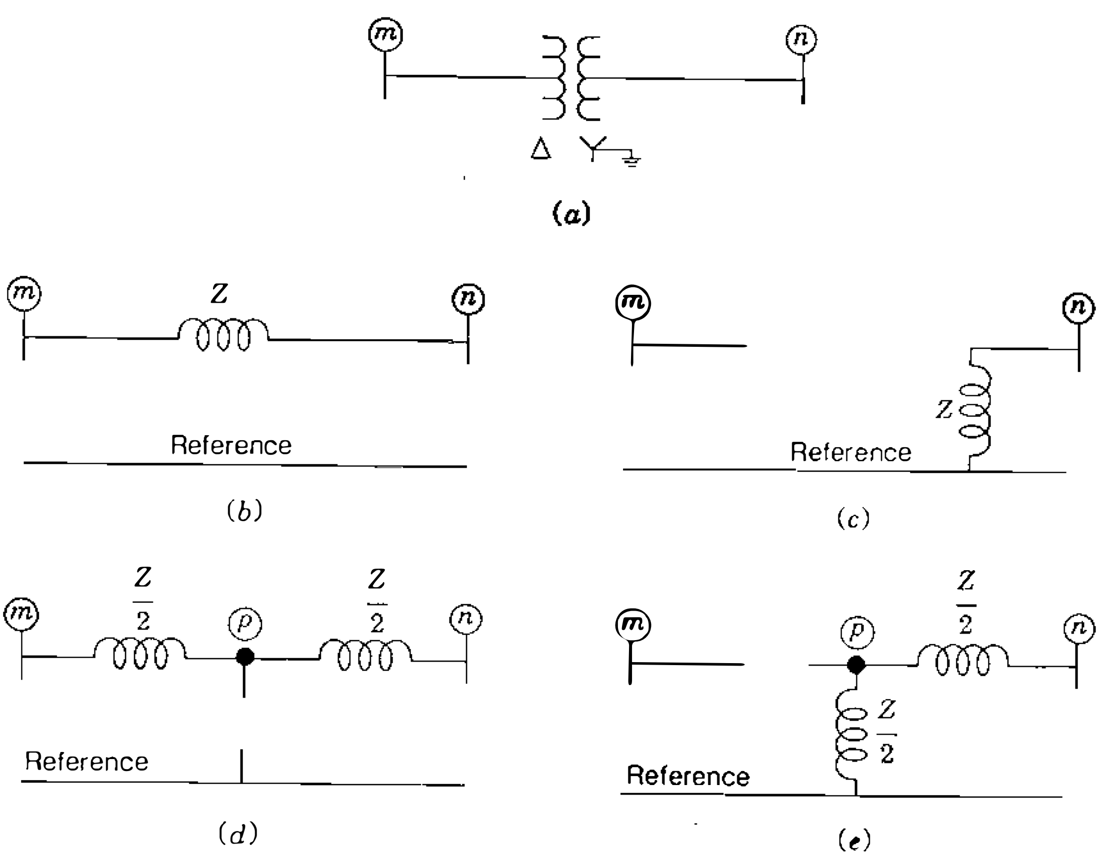


FIGURE 12.3
(a) Δ-Y grounded transformer with leakage impedance Z; (b) positive-sequence circuit; (c) zero-sequence circuit; (d) positive-sequence circuit with internal node; (e) zero-sequence circuit with internal node.

the matrices $\mathbf{Z}_{\text{bus}}^{(1)}$ and $\mathbf{Z}_{\text{bus}}^{(2)}$ in the usual manner—that is, by adding the negative of the leakage impedance Z between buses ⓜ and ⓝ in the positive- and negative-sequence networks. However, a similar strategy does not apply to the zero-sequence matrix $\mathbf{Z}_{\text{bus}}^{(0)}$ if it has been formed directly from the pictorial representation shown in Fig. 12.3(c). Adding $-Z$ between buses ⓜ and ⓝ does not remove the zero-sequence connection from bus ⓝ. To permit uniform procedures for all sequence networks, one strategy is to include an internal node ⓟ, as shown in Figs. 12.3(d) and 12.3(e).[1] Note that the leakage impedance is now subdivided into two parts between node ⓟ and the other nodes as shown. Connecting $-Z/2$ between buses ⓝ and ⓟ in each of the sequence circuits of Figs. 12.3(d) and 12.3(e) will open the transformer connections to bus ⓝ. Also, the open circuits can be represented within the computer algorithm by branches of arbitrarily large impedances (say, 10^6 per unit). Internal nodes of transformers can be useful in practical computer applications

[1]See H. E. Brown, *Solution of Large Networks by Matrix Methods*, 2d ed., John Wiley & Sons, Inc., New York, 1985.

of the $\mathbf{Z}_{\text{bus}}$ building algorithm. The reader is referred to the reference cited in footnote 1 for further guidance in handling open-circuit and short-circuit (bus tie) branches.

The faults to be discussed in succeeding sections may involve impedance Z_f between lines and from one or two lines to ground. When $Z_f = 0$, we have a direct short circuit, which is called a *bolted fault*. Although such direct short circuits result in the highest value of fault current and are therefore the most conservative values to use when determining the effects of anticipated faults, the fault impedance is seldom zero. Most faults are the result of insulator flashovers, where the impedance between the line and ground depends on the resistance of

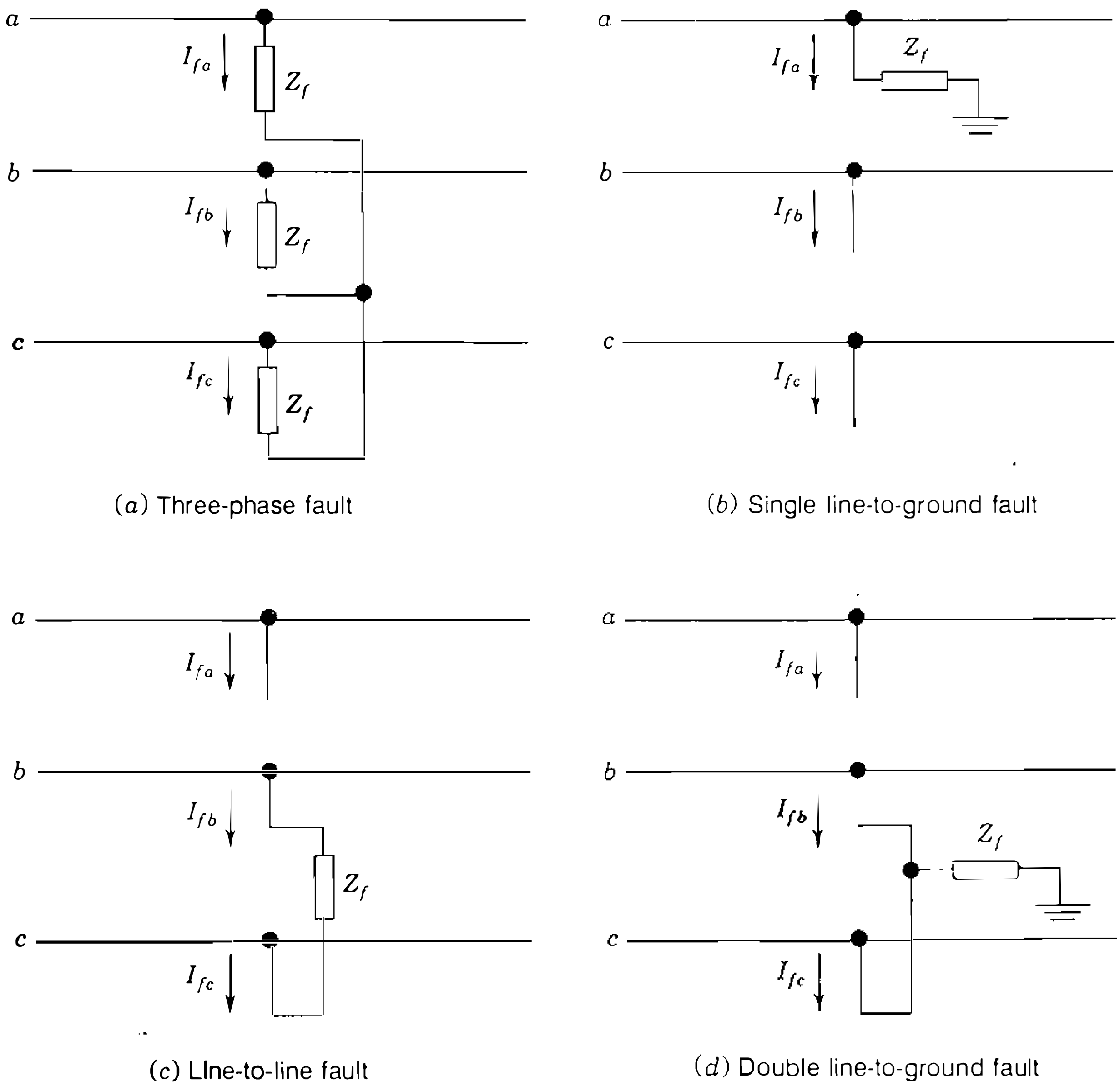


FIGURE 12.4
Connection diagrams of the hypothetical stubs for various faults through impedance.

the arc, of the tower itself, and of the tower footing if ground wires are not used. Tower-footing resistances form the major part of the resistance between line and ground and depend on the soil conditions. The resistance of dry earth is 10 to 100 times the resistance of swampy ground. Connections of the hypothetical stubs for faults through impedance Z_f are shown in Fig. 12.4.

A balanced system remains symmetrical after the occurrence of a *three-phase fault* having the same impedance between each line and a common point. Only positive-sequence currents flow. With the fault impedance Z_f equal in all phases, as shown in Fig. 12.4(a), we simply add impedance Z_f to the usual (positive-sequence) Thévenin equivalent circuit of the system at the fault bus ⓚ and calculate the fault current from the equation

$$I_{fa}^{(1)} = \frac{V_f}{Z_{kk}^{(1)} + Z_f} \tag{12.8}$$

For each of the other faults shown in Fig. 12.4, formal derivations of the equations for the symmetrical-component currents $I_{fa}^{(0)}$, $I_{fa}^{(1)}$, and $I_{fa}^{(2)}$ are provided in the sections which follow. In each case the fault point P is designated as bus ⓚ.

Example 12.1. Two synchronous machines are connected through three-phase transformers to the transmission line shown in Fig. 12.5. The ratings and reactances of the machines and transformers are

Machines 1 and 2: 100 MVA, 20 kV; $X_d'' = X_1 = X_2 = 20\%$,

$X_0 = 4\%$, $X_n = 5\%$

Transformers T_1 and T_2: 100 MVA, 20Δ/345Y kV; $X = 8\%$

On a chosen base of 100 MVA, 345 kV in the transmission-line circuit the line reactances are $X_1 = X_2 = 15\%$ and $X_0 = 50\%$. Draw each of the three sequence networks and find the zero-sequence bus impedance matrix by means of the $\mathbf{Z}_{\text{bus}}$ building algorithm.

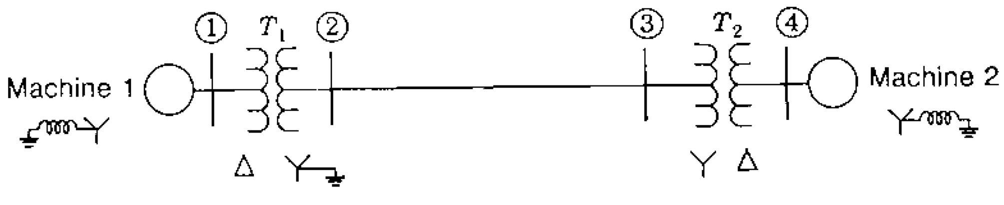


FIGURE 12.5
Single-line diagram of the system of Example 12.1.

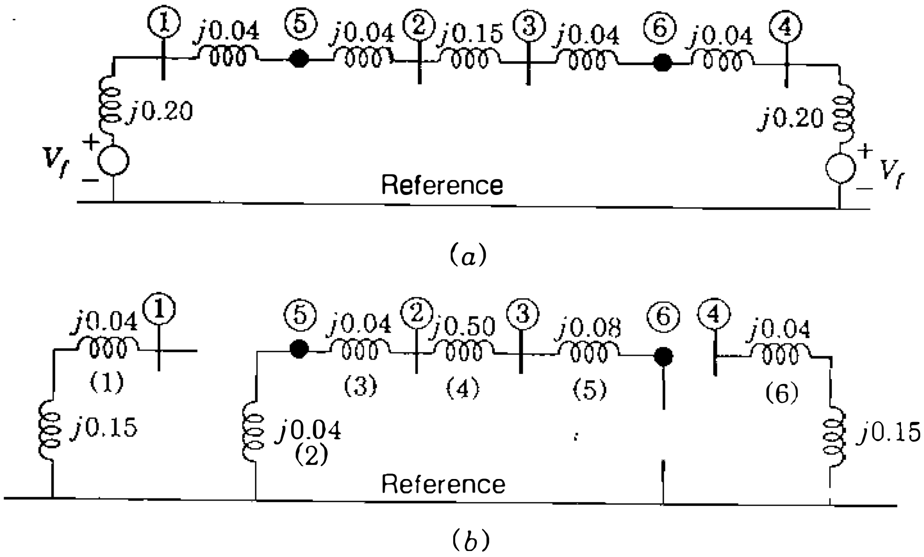


FIGURE 12.6
(*a*) Positive-sequence and (*b*) zero-sequence networks of the system of Fig. 12.5. Buses ⑤ and ⑥ are internal nodes of the transformers.

Solution. The given per-unit impedance values correspond to the chosen base, and so they can be used directly to form the sequence networks. Figure 12.6(*a*) shows the positive-sequence network, which is identical to the negative-sequence network when the emfs are short-circuited; Fig. 12.6(*b*) shows the zero-sequence network with reactance $3X_n = 0.15$ per unit in the neutral connection of each machine. Note that each transformer is assigned an internal node—bus ⑤ for transformer T_1 and bus ⑥ for transformer T_2. These internal nodes do not have an active role in the analysis of the system. In order to apply the $\mathbf{Z}_{\text{bus}}$ building algorithm, which is particularly simple in this example, let us label the zero-sequence branches from 1 to 7 as shown.

Step 1

Add branch 1 to the reference node

$$\begin{array}{cc} & ① \\ ① & \begin{bmatrix} j0.19 \end{bmatrix} \end{array}$$

Step 2

Add branch 2 to the reference node

$$\begin{array}{cc} & \begin{array}{cc} ① & ⑤ \end{array} \\ \begin{array}{c} ① \\ ⑤ \end{array} & \left[\begin{array}{c|c} j0.19 & 0 \\ \hline 0 & j0.04 \end{array}\right] \end{array}$$

Step 3

Add branch 3 between buses 5 and 2

$$\begin{array}{c} \\ 1 \\ 5 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 5 & 2 \end{array} \\ \left[\begin{array}{cc|c} j0.19 & 0 & 0 \\ 0 & j0.04 & j0.04 \\ \hline 0 & j0.04 & j0.08 \end{array}\right] \end{array}$$

Step 4

Add branch 4 between buses 2 and 3

$$\begin{array}{c} \\ 1 \\ 5 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 5 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc|c} j0.19 & 0 & 0 & 0 \\ 0 & j0.04 & j0.04 & j0.04 \\ 0 & j0.04 & j0.08 & j0.08 \\ \hline 0 & j0.04 & j0.08 & j0.58 \end{array}\right] \end{array}$$

Step 5

Add branch 5 between buses 3 and 6

$$\begin{array}{c} \\ 1 \\ 5 \\ 2 \\ 3 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 5 & 2 & 3 & 6 \end{array} \\ \left[\begin{array}{cccc|c} j0.19 & 0 & 0 & 0 & 0 \\ 0 & j0.04 & j0.04 & j0.04 & j0.04 \\ 0 & j0.04 & j0.08 & j0.08 & j0.08 \\ 0 & j0.04 & j0.08 & j0.58 & j0.58 \\ \hline 0 & j0.04 & j0.08 & j0.58 & j0.66 \end{array}\right] \end{array}$$

Step 6

Add branch 6 from bus 4 to the reference

$$\begin{array}{c} \\ 1 \\ 5 \\ 2 \\ 3 \\ 6 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 5 & 2 & 3 & 6 & 4 \end{array} \\ \left[\begin{array}{ccccc|c} j0.19 & 0 & 0 & 0 & 0 & 0 \\ 0 & j0.04 & j0.04 & j0.04 & j0.04 & 0 \\ 0 & j0.04 & j0.08 & j0.08 & j0.08 & 0 \\ 0 & j0.04 & j0.08 & j0.58 & j0.58 & 0 \\ 0 & j0.04 & j0.08 & j0.58 & j0.66 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & j0.19 \end{array}\right] \end{array}$$

Buses 5 and 6 are the fictitious internal nodes of the transformers which facilitate computer application of the $\mathbf{Z}_{\text{bus}}$ building algorithm. We have not shown calculations for the very high impedance branches representing the open circuits. Let us remove the rows and columns for buses 5 and 6 from the matrix to obtain the effective working matrix

$$
\mathbf{Z}_{\text{bus}}^{(0)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c} \begin{array}{cccc} \quad 1 \quad & \quad \bullet \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\
\begin{bmatrix} j0.19 & 0 & 0 & 0 \\ 0 & j0.08 & j0.08 & 0 \\ 0 & j0.08 & j0.58 & 0 \\ 0 & 0 & 0 & j0.19 \end{bmatrix} \end{array}
$$

The zeros in $\mathbf{Z}_{\text{bus}}^{(0)}$ show that zero-sequence current injected into bus 1 or bus 4 of Fig. 12.6(b) cannot cause voltages at the other buses because of the open circuits introduced by the Δ-Y transformers. Note also that the $j0.08$ per-unit reactance in series with the open circuit between buses 6 and 4 does not affect $\mathbf{Z}_{\text{bus}}^{(0)}$ since it cannot carry current.

By applying the $\mathbf{Z}_{\text{bus}}$ building algorithm to the positive- and negative-sequence networks in a similar manner, we obtain

$$
\mathbf{Z}_{\text{bus}}^{(1)} = \mathbf{Z}_{\text{bus}}^{(2)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c} \begin{array}{cccc} \quad 1 \quad & \quad \bullet \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\
\begin{bmatrix} j0.1437 & j0.1211 & j0.0789 & j0.0563 \\ j0.1211 & j0.1696 & j0.1104 & j0.0789 \\ j0.0789 & j0.1104 & j0.1696 & j0.1211 \\ j0.0563 & j0.0789 & j0.1211 & j0.1437 \end{bmatrix} \end{array}
$$

We use the above matrices in the examples which follow.

12.2 SINGLE LINE-TO-GROUND FAULTS

The single line-to-ground fault, the most common type, is caused by lightning or by conductors making contact with grounded structures. For a single line-to-ground fault through impedance Z_f the hypothetical stubs on the three lines are connected, as shown in Fig. 12.7, where phase a is the one on which the fault occurs. The relations to be developed for this type of fault will apply only when the fault is on phase a, but this should cause no difficulty since the phases are labeled arbitrarily and any phase can be designated as phase a. The conditions at the fault bus k are expressed by the following equations:

$$I_{fb} = 0 \qquad I_{fc} = 0 \qquad V_{ka} = Z_f I_{fa} \tag{12.9}$$

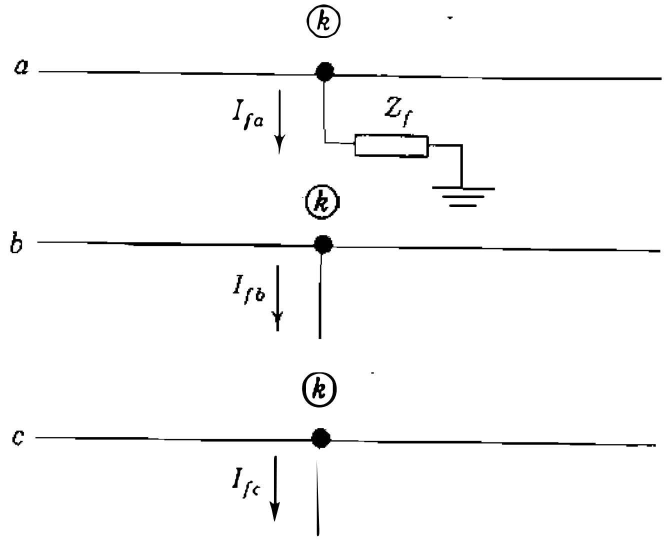


FIGURE 12.7
Connection diagram of the hypothetical stubs for a single line-to-ground fault. The fault point is called bus ⓚ.

With $I_{fb} = I_{fc} = 0$, the symmetrical components of the stub currents are given by

$$\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix}$$

and performing the multiplication yields

$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{I_{fa}}{3} \tag{12.10}$$

Substituting $I_{fa}^{(0)}$ for $I_{fa}^{(1)}$ and $I_{fa}^{(2)}$ shows that $I_{fa} = 3I_{fa}^{(0)}$, and from Eqs. (12.7) we obtain

$$\begin{aligned} V_{ka}^{(0)} &= - Z_{kk}^{(0)} I_{fa}^{(0)} \\ V_{ka}^{(1)} &= V_f - Z_{kk}^{(1)} I_{fa}^{(0)} \\ V_{ka}^{(2)} &= - Z_{kk}^{(2)} I_{fa}^{(0)} \end{aligned} \tag{12.11}$$

Summing these equations and noting that $V_{ka} = 3Z_f I_{fa}^{(0)}$ give

$$V_{ka} = V_{ka}^{(0)} + V_{ka}^{(1)} + V_{ka}^{(2)} = V_f - \left(Z_{kk}^{(0)} + Z_{kk}^{(1)} + Z_{kk}^{(2)}\right) I_{fa}^{(0)} = 3Z_f I_{fa}^{(0)}$$

Solving for $I_{fa}^{(0)}$ and combining the result with Eq. (12.10), we obtain

$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f} \tag{12.12}$$

Equations (12.12) are the fault current equations particular to the single line-to-ground fault through impedance Z_f, and they are used with the symmetrical-component relations to determine all the voltages and currents at the fault point P. If the Thévenin equivalent circuits of the three sequence networks of the system are connected *in series*, as shown in Fig. 12.8, we see that the currents and voltages resulting therefrom satisfy the above equations—for the Thévenin impedances looking into the three sequence networks at fault bus ⓚ are then in series with the fault impedance $3Z_f$ and the prefault voltage source V_f. With the equivalent circuits so connected, the voltage across each sequence network is the corresponding symmetrical component of the voltage V_{ka} at the fault bus ⓚ, and the current injected into each sequence network at bus ⓚ is the *negative* of the corresponding sequence current in the fault. The series connection of the Thévenin equivalents of the sequence networks, as shown in Fig. 12.8, is a convenient means of remembering the equations for the solution of the single line-to-ground fault, for all the necessary equations for the *fault*

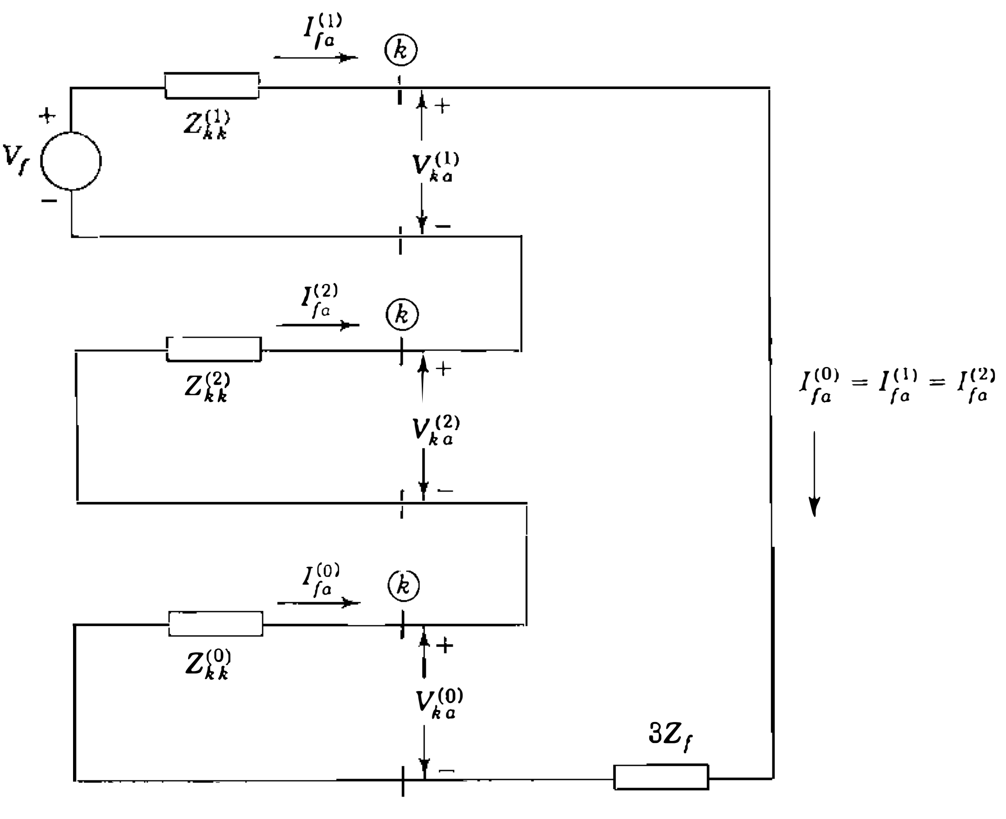


FIGURE 12.8
Connection of the Thévenin equivalents of the sequence networks to simulate a single line-to-ground fault on phase a at bus ⓚ of the system.

point can be determined from the sequence-network connection. Once the currents $I_{fa}^{(0)}$, $I_{fa}^{(1)}$, and $I_{fa}^{(2)}$ are known, the components of voltages at all other buses of the system can be determined from the bus impedance matrices of the sequence networks according to Eqs. (12.6).

Example 12.2. Two synchronous machines are connected through three-phase transformers to the transmission line shown in Fig. 12.9(*a*). The ratings and reactances of the machines and transformers are

Machines 1 and 2: 100 MVA, 20 kV; $X_d'' = X_1 = X_2 = 20\%$,

$X_0 = 4\%$, $X_n = 5\%$

Transformers T_1 and T_2: 100 MVA, 20Y/345Y kV; $X = 8\%$

Both transformers are solidly grounded on two sides. On a chosen base of 100 MVA, 345 kV in the transmission-line circuit the line reactances are $X_1 = X_2 = 15\%$ and $X_0 = 50\%$. The system is operating at nominal voltage without prefault currents when a bolted ($Z_f = 0$) single line-to-ground fault occurs on phase A at bus ③. Using the bus impedance matrix for each of the three sequence networks, determine the subtransient current to ground at the fault, the line-to-ground voltages at the terminals of machine 2, and the subtransient current out of phase c of machine 2.

Solution. The system is the same as in Example 12.1, except that the transformers are now Y-Y connected. Therefore, we can continue to use $\mathbf{Z}_{\text{bus}}^{(1)}$ and $\mathbf{Z}_{\text{bus}}^{(2)}$ corresponding to Fig. 12.6(*a*), as given in Example 12.1. However, because the

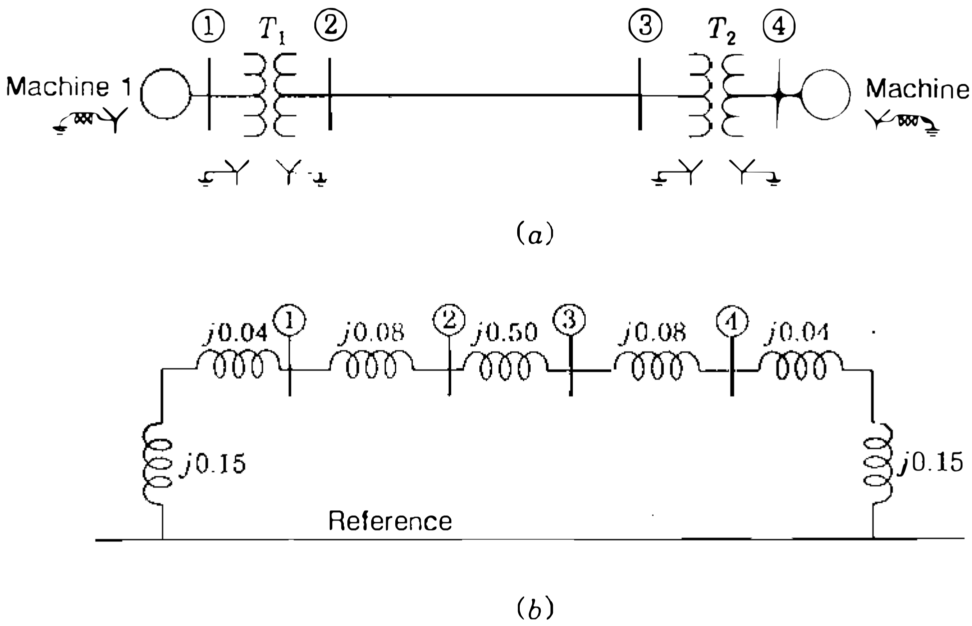


FIGURE 12.9
(*a*) The single-line diagram and (*b*) zero-sequence network of the system of Example 12.2.

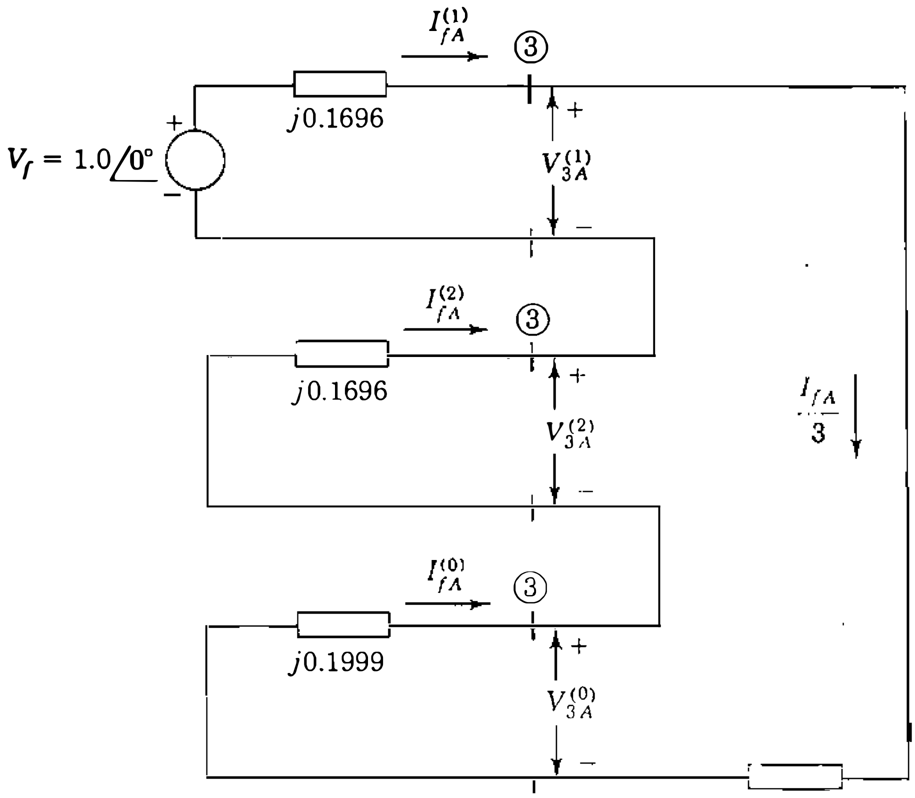


FIGURE 12.10
Series connection of the Thévenin equivalents of the sequence networks for the single line-to-ground fault of Example 12.2.

transformers are solidly grounded on both sides, the zero-sequence network is fully connected, as shown in Fig. 12.9(*b*), and has the bus impedance matrix

$$\mathbf{Z}_{\text{bus}}^{(0)} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccc} \quad ① \quad & \quad ② \quad & \quad ③ \quad & \quad ④ \quad \end{array} \\ \begin{bmatrix} j0.1553 & j0.1407 & j0.0493 & j0.0347 \\ j0.1407 & j0.1999 & j0.0701 & j0.0493 \\ j0.0493 & j0.0701 & j0.1999 & j0.1407 \\ j0.0347 & j0.0493 & j0.1407 & j0.1553 \end{bmatrix} \end{array}$$

Since the line-to-ground fault is at bus ③, we must connect the Thévenin equivalent circuits of the sequence networks in series, as shown in Fig. 12.10. From this figure we can calculate the symmetrical components of the current I_{fA} out of the system and into the fault,

$$I_{fA}^{(0)} = I_{fA}^{(1)} = I_{fA}^{(2)} = \frac{V_f}{Z_{33}^{(1)} + Z_{33}^{(2)} + Z_{33}^{(0)}}$$

$$= \frac{1.0\angle 90°}{j(0.1696 + 0.1696 + 0.1999)} = -j1.8549 \text{ per unit}$$

The total current in the fault is

$$I_{fA} = 3I_{fA}^{(0)} = -j5.5648 \text{ per unit}$$

and since the base current in the high-voltage transmission line is $100{,}000/\sqrt{3} \times 345 = 167.35$ A, we have

$$I_{fA} = -j5.5648 \times 167.35 = 931\underline{/270^\circ} \text{ A}$$

The phase-a sequence voltages at bus ④, the terminals of machine 2, are calculated from Eqs. (12.6) with $k = 3$ and $j = 4$,

$$V_{4a}^{(0)} = \quad - Z_{43}^{(0)}I_{fA}^{(0)} = \quad - (j0.1407)(-j1.8549) = -0.2610 \text{ per unit}$$

$$V_{4a}^{(1)} = V_f - Z_{43}^{(1)}I_{fA}^{(1)} = 1 - (j0.1211)(-j1.8549) = \quad 0.7754 \text{ per unit}$$

$$V_{4a}^{(2)} = \quad - Z_{43}^{(2)}I_{fA}^{(2)} = \quad - (j0.1211)(-j1.8549) = -0.2246 \text{ per unit}$$

Note that subscripts A and a denote voltages and currents in the high-voltage and low-voltage circuits, respectively, of the Y-Y connected transformer. No phase shift is involved. From the above symmetrical components we can calculate a-b-c line-to-ground voltages at bus ④ as follows:

$$\begin{bmatrix} V_{4a} \\ V_{4b} \\ V_{4c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.2610 \\ 0.7754 \\ -0.2246 \end{bmatrix} = \begin{bmatrix} 0.2898 + j0.0 \\ -0.5364 - j0.8660 \\ -0.5364 + j0.8660 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2898\underline{/0^\circ} \\ 1.0187\underline{/-121.8^\circ} \\ 1.0187\underline{/121.8^\circ} \end{bmatrix}$$

To express the line-to-ground voltages of machine 2 in kilovolts, we multiply by $20/\sqrt{3}$, which gives

$$V_{4a} = 3.346\underline{/0^\circ} \text{ kV} \qquad V_{4b} = 11.763\underline{/-121.8^\circ} \text{ kV} \qquad V_{4c} = 11.763\underline{/121.8^\circ} \text{ kV}$$

To determine phase-c current out of machine 2, we must first calculate the symmetrical components of the phase-a current in the branches representing the machine in the sequence networks. From Fig. 12.9(b) the zero-sequence current *out* of the machine is

$$I_a^{(0)} = -\frac{V_{4a}^{(0)}}{jX_0} = \frac{0.2610}{j0.04} = -j6.525 \text{ per unit}$$

and from Fig. 12.6(*a*) the other sequence currents are calculated as

$$I_a^{(1)} = \frac{V_f - V_{4a}^{(1)}}{jX''} = \frac{1.0 - 0.7754}{j0.20} = -j1.123 \text{ per unit}$$

$$I_a^{(2)} = -\frac{V_{4a}^{(2)}}{jX_2} = \frac{0.2246}{j0.20} = -j1.123 \text{ per unit}$$

Note that the machine currents are shown without subscript f, which is reserved exclusively for the (stub) currents and voltages of the fault point. The phase-c currents in machine 2 are now easily calculated,

$$I_c = I_a^{(0)} + aI_a^{(1)} + a^2I_a^{(2)}$$

$$= -j6.525 + a(-j1.123) + a^2(-j1.123) = -j5.402 \text{ per unit}$$

The base current in the machine circuits is $100{,}000/(\sqrt{3} \times 20) = 2886.751$ A, and so $|I_c| = 15{,}594$ A. Other voltages and currents in the system can be calculated similarly.

12.3 LINE-TO-LINE FAULTS

To represent a line-to-line fault through impedance Z_f, the hypothetical stubs on the three lines at the fault are connected, as shown in Fig. 12.11. Bus ⓚ is again the fault point P, and without any loss of generality, the line-to-line fault is regarded as being on phases b and c. The following relations must be satisfied at the fault point

$$I_{fa} = 0 \qquad I_{fb} = -I_{fc} \qquad V_{kb} - V_{kc} = I_{fb}Z_f \tag{12.13}$$

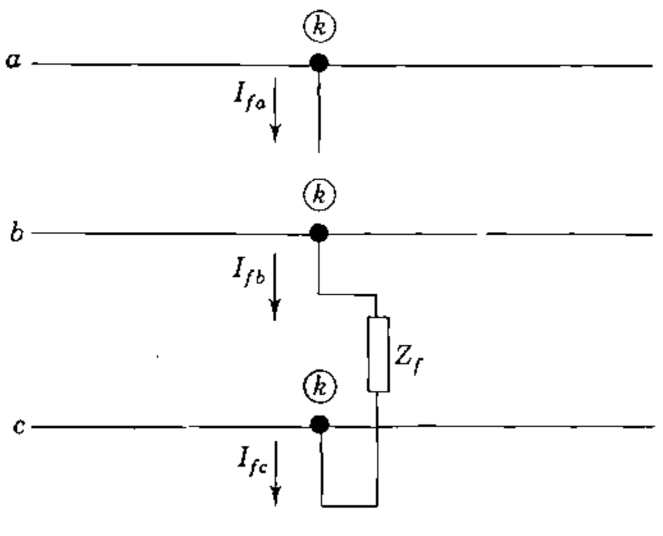


FIGURE 12.11
Connection of the hypothetical stubs for a line-to-line fault. The fault point is called bus ⓚ.

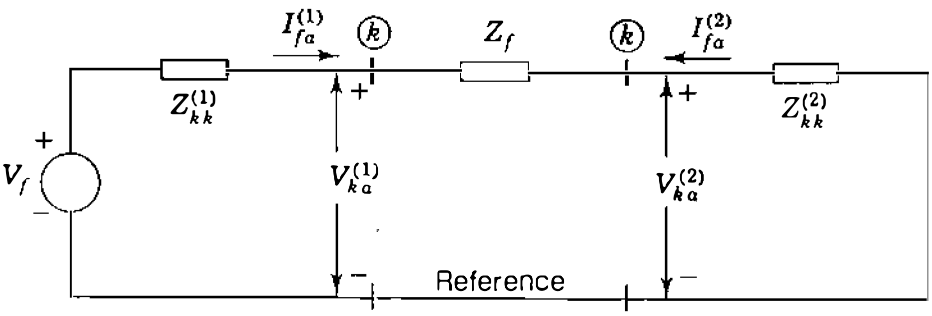


FIGURE 12.12
Connection of the Thévenin equivalents of the positive- and negative-sequence networks for a line-to-line fault between phases b and c at bus ⓚ of the system.

Since $I_{fb} = -I_{fc}$ and $I_{fa} = 0$, the symmetrical components of current are

$$\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix}$$

and multiplying through in this equation shows that

$$I_{fa}^{(0)} = 0 \tag{12.14}$$

$$I_{fa}^{(1)} = -I_{fa}^{(2)} \tag{12.15}$$

The voltages throughout the zero-sequence network must be zero since there are no zero-sequence sources, and because $I_{fa}^{(0)} = 0$, current is not being injected into that network due to the fault. Hence, line-to-line fault calculations do not involve the zero-sequence network, which remains the same as before the fault—a dead network.

To satisfy the requirement that $I_{fa}^{(1)} = -I_{fa}^{(2)}$, let us connect the Thévenin equivalents of the positive- and negative-sequence networks *in parallel*, as shown in Fig. 12.12. To show that this connection of the networks also satisfies the voltage equation $V_{kb} - V_{kc} = I_{fb}Z_f$, we now expand each side of that equation separately as follows:

$$V_{kb} - V_{kc} = \left(V_{kb}^{(1)} + V_{kb}^{(2)}\right) - \left(V_{kc}^{(1)} + V_{kc}^{(2)}\right) = \left(V_{kb}^{(1)} - V_{kc}^{(1)}\right) + \left(V_{kb}^{(2)} - V_{kc}^{(2)}\right)$$

$$= (a^2 - a)V_{ka}^{(1)} + (a - a^2)V_{ka}^{(2)} = (a^2 - a)\left(V_{ka}^{(1)} - V_{ka}^{(2)}\right)$$

$$I_{fb}Z_f = \left(I_{fb}^{(1)} + I_{fb}^{(2)}\right)Z_f = \left(a^2 I_{fa}^{(1)} + a I_{fa}^{(2)}\right)Z_f$$

Equating both terms and setting $I_{fa}^{(2)} = -I_{fa}^{(1)}$ as in Fig. 12.12, we obtain

$$(a^2 - a)\left(V_{ka}^{(1)} - V_{ka}^{(2)}\right) = (a^2 - a) I_{fa}^{(1)} Z_f$$

or

$$V_{ka}^{(1)} - V_{ka}^{(2)} = I_{fa}^{(1)} Z_f \tag{12.16}$$

which is precisely the voltage-drop equation for impedance Z_f in Fig. 12.12.

Thus, all the fault conditions of Eqs. (12.13) are satisfied by connecting the positive- and negative-sequence networks *in parallel* through impedance Z_f, as shown in Fig. 12.12. The zero-sequence network is inactive and does not enter into the line-to-line fault calculations. The equation for the positive-sequence current in the fault can be determined directly from Fig. 12.12 so that

$$I_{fa}^{(1)} = -I_{fa}^{(2)} = \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_f} \tag{12.17}$$

For a bolted line-to-line fault we set $Z_f = 0$.

Equations (12.17) are the fault current equations for a line-to-line fault through impedance Z_f. Once $I_{fa}^{(1)}$ and $I_{fa}^{(2)}$ are known, they can be treated as injections $-I_{fa}^{(1)}$ and $-I_{fa}^{(2)}$ into the positive- and negative-sequence networks, respectively, and the changes in the sequence voltages at the buses of the system due to the fault can be obtained from the bus impedance matrices, as previously demonstrated. When Δ-Y transformers are present, phase shift of the positive- and negative-sequence currents and voltages must be taken into account in the calculations. The following example shows how this is accomplished.

Example 12.3. The same system as in Example 12.1 is operating at nominal system voltage without prefault currents when a bolted line-to-line fault occurs at bus ③. Using the bus impedance matrices of the sequence networks for subtransient conditions, determine the currents in the fault, the line-to-line voltages at the fault bus, and the line-to-line voltages at the terminals of machine 2.

Solution. $\mathbf{Z}_{\text{bus}}^{(1)}$ and $\mathbf{Z}_{\text{bus}}^{(2)}$ are already set forth in Example 12.1. Although $\mathbf{Z}_{\text{bus}}^{(0)}$ is also given, we are not concerned with the zero-sequence network in this solution since the fault is line to line.

To simulate the fault, the Thévenin equivalent circuits at bus ③ of the positive- and negative-sequence networks of Example 12.1 are connected in parallel, as shown in Fig. 12.13. From this figure the sequence currents are calculated as follows:

$$I_{fA}^{(1)} = -I_{fA}^{(2)} = \frac{V_f}{Z_{33}^{(1)} + Z_{33}^{(2)}} = \frac{1 + j0}{j0.1696 + j0.1696} = -j2.9481 \text{ per unit}$$

Uppercase A is used here because the fault is in the high-voltage transmission-line

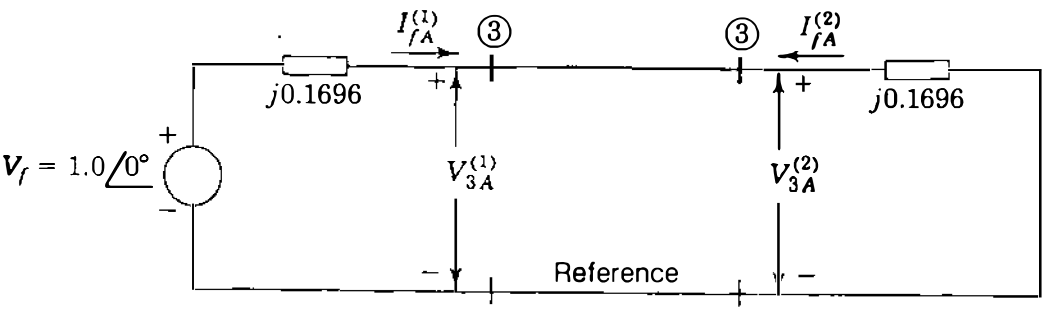


FIGURE 12.13
Connection of the Thévenin equivalent circuits for the line-to-line fault of Example 12.3.

circuit. Since $I_{fA}^{(0)} = 0$, the components of currents in the fault are calculated from

$$I_{fA} = I_{fA}^{(1)} + I_{fA}^{(2)} = -j2.9481 + j2.9481 = 0$$

$$I_{fB} = a^2 I_{fA}^{(1)} + a I_{fA}^{(2)} = -j2.9481(-0.5 - j0.866) + j2.9481(-0.5 + j0.866)$$

$$= -5.1061 + j0 \text{ per unit}$$

$$I_{fC} = -I_{fB} = 5.1061 + j0 \text{ per unit}$$

As in Example 12.2, base current in the transmission line is 167.35 A, and so

$$I_{fA} = 0$$

$$I_{fB} = -5.1061 \times 167.35 = 855\angle 180° \text{ A}$$

$$I_{fC} = -5.1061 \times 167.35 = 855\angle 0° \text{ A}$$

Symmetrical components of phase-A voltage to ground at bus ③ are

$$V_{3A}^{(0)} = 0$$

$$V_{3A}^{(1)} = V_{3A}^{(2)} = 1 - Z_{kk}^{(1)} I_{fA}^{(1)} = 1 - (j0.1696)(-j2.9481) = 0.5 + j0 \text{ per unit}$$

Line-to-ground voltages at fault bus ③ are

$$V_{3A} = V_{3A}^{(0)} + V_{3A}^{(1)} + V_{3A}^{(2)} = 0 + 0.5 + 0.5 = 1.0\angle 0° \text{ per unit}$$

$$V_{3B} = V_{3A}^{(0)} + a^2 V_{3A}^{(1)} + a V_{3A}^{(2)} = 0 + a^2 0.5 + a0.5 = 0.5\angle 180° \text{ per unit}$$

$$V_{3C} = V_{3B} = 0.5\angle 180° \text{ per unit}$$

Line-to-line voltages at fault bus ③ are

$$V_{3,AB} = V_{3A} - V_{3B} = (\ \ 1.0 + j0) - (-0.50 + j0) = 1.5\underline{/0^\circ} \text{ per unit}$$

$$V_{3,BC} = V_{3B} - V_{3C} = (-0.5 + j0) - (-0.50 + j0) = 0$$

$$V_{3,CA} = V_{3C} - V_{3A} = (-0.5 + j0) - (\ \ 1.0 + j0) = 1.5\underline{/180^\circ} \text{ per unit}$$

Expressed in volts, these line-to-line voltages are

$$V_{3,AB} = 1.5\underline{/0^\circ} \times \frac{345}{\sqrt{3}} = 299\underline{/0^\circ} \text{ kV}$$

$$V_{3,BC} = 0$$

$$V_{3,CA} = 1.5\underline{/180^\circ} \times \frac{345}{\sqrt{3}} = 299\underline{/180^\circ} \text{ kV}$$

For the moment, let us avoid phase shifts due to the Δ-Y transformer connected to machine 2 and proceed to calculate the sequence voltages of phase A at bus ④ using the bus impedance matrices of Example 12.1 and Eqs. (12.6) with $k = 3$ and $j = 4$

$$V_{4A}^{(0)} = \quad - Z_{43}^{(0)} I_{fA}^{(0)} = 0$$

$$V_{4A}^{(1)} = V_f - Z_{43}^{(1)} I_{fA}^{(1)} = 1 - (j0.1211)(-j2.9481) = 0.643 \text{ per unit}$$

$$V_{4A}^{(2)} = \quad - Z_{43}^{(2)} I_{fA}^{(2)} = \quad - (j0.1211)(j2.9481) = 0.357 \text{ per unit}$$

To account for phase shifts in stepping *down* from the high-voltage transmission line to the low-voltage terminals of machine 2, we must retard the positive-sequence voltage and advance the negative-sequence voltage by 30°. At machine 2 terminals, indicated by lowercase a, the voltages are

$$V_{4a}^{(0)} = 0$$

$$V_{4a}^{(1)} = V_{4A}^{(1)}\underline{/-30^\circ} = 0.643\underline{/-30^\circ} = 0.5569 - j0.3215 \text{ per unit}$$

$$V_{4a}^{(2)} = V_{4A}^{(2)}\underline{/30^\circ} = 0.357\underline{/30^\circ} = 0.3092 + j0.1785 \text{ per unit}$$

$$V_{4a} = V_{4a}^{(0)} + V_{4a}^{(1)} + V_{4a}^{(2)} = 0 + (0.5569 - j0.3215) + (0.3092 + j0.1785)$$

$$= 0.8661 - j0.1430 = 0.8778\underline{/-9.4^\circ} \text{ per unit}$$

Phase-b voltages at terminals of machine 2 are now calculated,

$$V_{4b}^{(0)} = V_{4a}^{(0)} \quad = 0$$

$$V_{4b}^{(1)} = a^2 V_{4a}^{(1)} = \left(1\angle 240^\circ\right)\left(0.643\angle -30^\circ\right) = -0.5569 - j0.3215 \text{ per unit}$$

$$V_{4b}^{(2)} = a\ V_{4a}^{(2)} = \left(1\angle 120^\circ\right)\left(0.357\angle 30^\circ\right) \quad = -0.3092 + j0.1785 \text{ per unit}$$

$$V_{4b} = V_{4b}^{(0)} + V_{4b}^{(1)} + V_{4b}^{(2)} = 0 + (-0.5569 - j0.3215) + (-0.3092 + j0.1785)$$

$$= -0.8661 - j0.143 = 0.8778\angle -170.6^\circ \text{ per unit}$$

and for phase c of machine 2

$$V_{4c}^{(0)} = V_{4a}^{(0)} = 0$$

$$V_{4c}^{(1)} = a\ V_{4a}^{(1)} = \left(1\angle 120^\circ\right)\left(0.643\angle -30^\circ\right) = 0.643\angle 90^\circ \text{ per unit}$$

$$V_{4c}^{(2)} = a^2 V_{4a}^{(2)} = \left(1\angle 240^\circ\right)\left(0.357\angle 30^\circ\right) \quad = 0.357\angle -90^\circ \text{ per unit}$$

$$V_{4c} = V_{4c}^{(0)} + V_{4c}^{(1)} + V_{4c}^{(2)} = 0 + (j0.643) + (-j0.357) = 0 + j0.286 \text{ per unit}$$

Line-to-line voltages at the terminals of machine 2 are

$$V_{4,ab} = V_{4a} - V_{4b} = (0.8661 - j0.143) - (0.8661 - j0.143)$$

$$= 1.7322 + j0 \text{ per unit}$$

$$V_{4,bc} = V_{4b} - V_{4c} = (-0.8661 - j0.143) - (0 + j0.286)$$

$$= -0.8661 - j0.429 = 0.9665\angle -153.65^\circ \text{ per unit}$$

$$V_{4,ca} = V_{4c} - V_{4a} = (0 + j0.286) - (0.8661 - j0.143)$$

$$= -0.8661 + j0.429 = 0.9665\angle 153.65^\circ \text{ per unit}$$

In volts, line-to-line voltages at machine 2 terminals are

$$V_{4,ab} = 1.7322\angle 0^\circ \qquad \times \frac{20}{\sqrt{3}} = 20\angle 0^\circ \text{ kV}$$

$$V_{4,bc} = 0.9665\angle -153.65^\circ \times \frac{20}{\sqrt{3}} = 11.2\angle -153.65^\circ \text{ kV}$$

$$V_{4,ca} = 0.9665\angle 153.65^\circ \quad \times \frac{20}{\sqrt{3}} = 11.2\angle 153.65^\circ \text{ kV}$$

Thus, from the currents $I_{fA}^{(0)}$, $I_{fA}^{(1)}$, and $I_{fA}^{(2)}$ of the fault and the bus impedance matrices of the sequence networks we can determine the unbalanced bus voltages and branch currents throughout the system due to the line-to-line fault.

12.4 DOUBLE LINE-TO-GROUND FAULTS

For a double line-to-ground fault the hypothetical stubs are connected, as shown in Fig. 12.14. Again, the fault is taken to be on phases b and c, and the relations now existing at the fault bus ⓚ are

$$I_{fa} = 0 \qquad V_{kb} = V_{kc} = (I_{fb} + I_{fc})Z_f \tag{12.18}$$

Since I_{fa} is zero, the zero-sequence current is given by $I_{fa}^{(0)} = (I_{fb} + I_{fc})/3$, and the voltages of Eq. (12.18) then become

$$V_{kb} = V_{kc} = 3Z_f I_{fa}^{(0)} \tag{12.19}$$

Substituting V_{kb} for V_{kc} in the symmetrical-component transformation, we find that

$$\begin{bmatrix} V_{ka}^{(0)} \\ V_{ka}^{(1)} \\ V_{ka}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix} \tag{12.20}$$

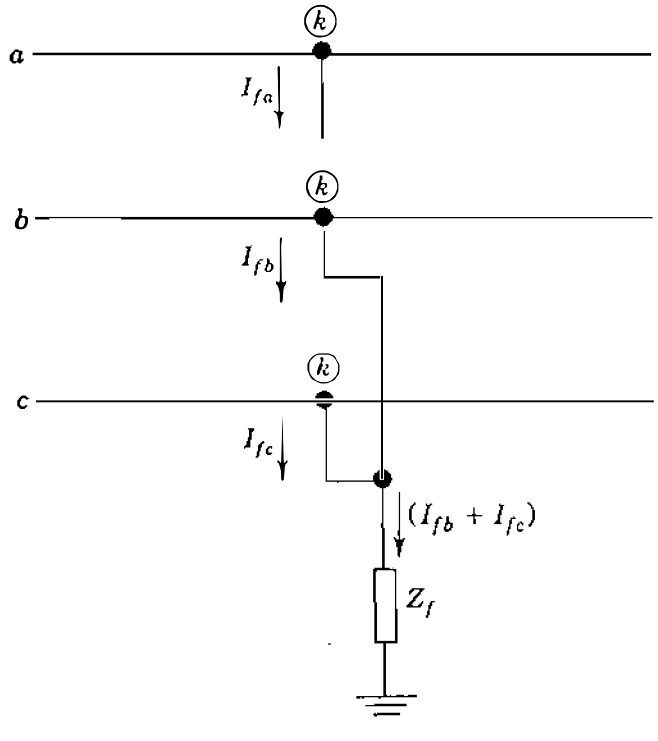


FIGURE 12.14
Connection diagram for the hypothetical stubs for a double line-to-ground fault. The fault point is called bus ⓚ.

The second and third rows of this equation show that

$$V_{ka}^{(1)} = V_{ka}^{(2)} \tag{12.21}$$

while the first row and Eq. (12.19) show that

$$3V_{ka}^{(0)} = V_{ka} + 2V_{kb} = \left(V_{ka}^{(0)} + V_{ka}^{(1)} + V_{ka}^{(2)}\right) + 2\left(3Z_f I_{fa}^{(0)}\right)$$

Collecting zero-sequence terms on one side, setting $V_{ka}^{(2)} = V_{ka}^{(1)}$, and solving for $V_{ka}^{(1)}$, we obtain

$$V_{ka}^{(1)} = V_{ka}^{(0)} - 3Z_f I_{fa}^{(0)} \tag{12.22}$$

Bringing together Eqs. (12.21) and (12.22), and again noting that $I_{fa} = 0$, we arrive at the results

$$V_{ka}^{(1)} = V_{ka}^{(2)} = V_{ka}^{(0)} - 3Z_f I_{fa}^{(0)}$$

$$I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = 0 \tag{12.23}$$

These characterizing equations of the double line-to-ground fault are satisfied when all three of the sequence networks are connected *in parallel*, as shown in Fig. 12.15. The diagram of network connections shows that the positive-sequence current $I_{fa}^{(1)}$ is determined by applying prefault voltage V_f across the total impedance consisting of $Z_{kk}^{(1)}$ in series with the parallel combination of $Z_{kk}^{(2)}$ and $(Z_{kk}^{(0)} + 3Z_f)$. That is,

$$I_{fa}^{(1)} = \frac{V_f}{Z_{kk}^{(1)} + \left[\dfrac{Z_{kk}^{(2)}\left(Z_{kk}^{(0)} + 3Z_f\right)}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}\right]} \tag{12.24}$$

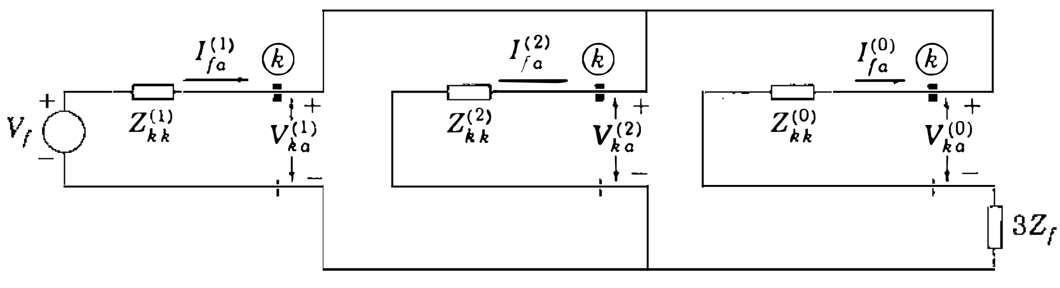


FIGURE 12.15
Connection of the Thévenin equivalents of the sequence networks for a double line-to-ground fault on phases *b* and *c* at bus (k) of the system.

The negative- and zero-sequence currents *out* of the system and *into* the fault can be determined from Fig. 12.15 by simple current division so that

$$I_{fa}^{(2)} = -I_{fa}^{(1)}\left[\frac{Z_{kk}^{(0)} + 3Z_f}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}\right] \tag{12.25}$$

$$I_{fa}^{(0)} = -I_{fa}^{(1)}\left[\frac{Z_{kk}^{(2)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}\right] \tag{12.26}$$

For a bolted fault Z_f is set equal to 0 in the above equations. When $Z_f = \infty$, the zero-sequence circuit becomes an open circuit; no zero-sequence current can then flow and the equations revert back to those for the line-to-line fault discussed in the preceding section.

Again, we observe that the sequence currents $I_{fa}^{(1)}$, $I_{fa}^{(2)}$, and $I_{fa}^{(0)}$, once calculated, can be treated as negative injections into the sequence networks at the fault bus ⓚ and the sequence voltage *changes* at all buses of the system can then be calculated from the bus impedance matrices, as we have done in preceding sections.

Example 12.4. Find the subtransient currents and the line-to-line voltages at the fault under subtransient conditions when a double line-to-ground fault with $Z_f = 0$ occurs at the terminals of machine 2 in the system of Fig. 12.5. Assume that the system is unloaded and operating at rated voltage when the fault occurs. Use the bus impedance matrices and neglect resistance.

Solution. The bus impedance matrices $\mathbf{Z}_{\text{bus}}^{(1)}$, $\mathbf{Z}_{\text{bus}}^{(2)}$, and $\mathbf{Z}_{\text{bus}}^{(0)}$ are the same as in Example 12.1, and so the Thévenin impedances at fault bus ④ are equal in per unit to the diagonal elements $Z_{44}^{(0)} = j0.19$ and $Z_{44}^{(1)} = Z_{44}^{(2)} = j0.1437$. To simulate the double line-to-ground fault at bus ④, we connect the Thévenin equivalents of all three sequence networks in parallel, as shown in Fig. 12.16, from which we

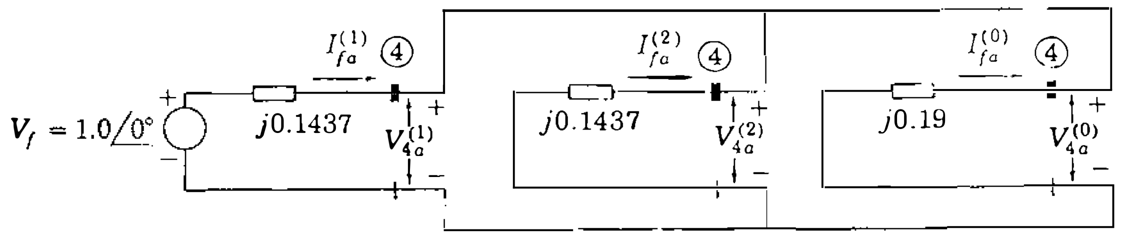


FIGURE 12.16
Connection of the Thévenin equivalents of the sequence networks for the double line-to-line fault of Example 12.4.

obtain

$$I_{fa}^{(1)} = \frac{V_f}{Z_{44}^{(1)} + \left[\dfrac{Z_{44}^{(2)}Z_{44}^{(0)}}{Z_{44}^{(2)} + Z_{44}^{(0)}}\right]} = \frac{1 + j0}{j0.1437 + \left[\dfrac{(j0.1437)(j0.19)}{(j0.1437 + j0.19)}\right]}$$

$$= -j4.4342 \text{ per unit}$$

Therefore, the sequence voltages at the fault are

$$V_{4a}^{(1)} = V_{4a}^{(2)} = V_{4a}^{(0)} = V_f - I_{fa}^{(1)}Z_{44}^{(1)} = 1 - (-j4.4342)(j0.1437) = 0.3628 \text{ per unit}$$

Current injections into the negative- and zero-sequence networks at the fault bus are calculated by current division as follows:

$$I_{fa}^{(2)} = -I_{fa}^{(1)}\left[\frac{Z_{44}^{(0)}}{Z_{44}^{(2)} + Z_{44}^{(0)}}\right] = j4.4342\left[\frac{j0.19}{j(0.1437 + 0.19)}\right] = j2.5247 \text{ per unit}$$

$$I_{fa}^{(0)} = -I_{fa}^{(1)}\left[\frac{Z_{44}^{(2)}}{Z_{44}^{(2)} + Z_{44}^{(0)}}\right] = j4.4342\left[\frac{j0.1437}{j(0.1437 + 0.19)}\right] = j1.9095 \text{ per unit}$$

The currents out of the system at the fault point are

$$I_{fa} = I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = j1.9095 - j4.4342 + j2.5247 = 0$$

$$I_{fb} = I_{fa}^{(0)} + a^2 I_{fa}^{(1)} + a I_{fa}^{(2)}$$

$$= j1.9095 + (1\angle 240^\circ)(4.4342\angle -90^\circ) + (1\angle 120^\circ)(2.5247\angle 90^\circ)$$

$$= -6.0266 + j2.8642 = 6.6726\angle 154.6^\circ \text{ per unit}$$

$$I_{fc} = I_{fa}^{(0)} + a I_{fa}^{(1)} + a^2 I_{fa}^{(2)}$$

$$= j1.9095 + (1\angle 120^\circ)(4.4342\angle -90^\circ) + (1\angle 240^\circ)(2.5247\angle 90^\circ)$$

$$= 6.0266 + j2.8642 = 6.6726\angle 25.4^\circ \text{ per unit}$$

and the current I_f into the ground is

$$I_f = I_{fb} + I_{fc} = 3I_{fa}^{(0)} = j5.7285 \text{ per unit}$$

Calculating *a-b-c* voltages at the fault bus, we find that

$$V_{4a} = V_{4a}^{(0)} + V_{4a}^{(1)} + V_{4a}^{(2)} = 3V_{4a}^{(1)} = 3(0.3628) = 1.0884 \text{ per unit}$$

$$V_{4b} = V_{4c} = 0$$

$$V_{4,ab} = V_{4a} - V_{4b} = 1.0884 \text{ per unit}$$

$$V_{4,bc} = V_{4b} - V_{4c} = 0$$

$$V_{4,ca} = V_{4c} - V_{4a} = -1.0884 \text{ per unit}$$

Base current equals $100 \times 10^3/(\sqrt{3} \times 20) = 2887$ A in the circuit of machine 2, and so we find that

$$I_{fa} = 0$$

$$I_{fb} = 2887 \times 6.6726\underline{/154.6^\circ} = 19{,}262\underline{/154.6^\circ} \text{ A}$$

$$I_{fc} = 2887 \times 6.6726\underline{/25.4^\circ} = 19{,}262\underline{/25.4^\circ} \text{ A}$$

$$I_f = 2887 \times 5.7285\underline{/90^\circ} = 16{,}538\underline{/90^\circ} \text{ A}$$

The base line-to-neutral voltage in machine 2 is $20/\sqrt{3}$ kV, and so

$$V_{4,ab} = 1.0884 \times \frac{20}{\sqrt{3}} = 12.568\underline{/0^\circ} \text{ kV}$$

$$V_{4,bc} = 0$$

$$V_{4,ca} = -1.0884 \times \frac{20}{\sqrt{3}} = 12.568\underline{/180^\circ} \text{ kV}$$

Examples 12.3 and 12.4 show that phase shifts due to Δ-Y transformers do not enter into the calculations of sequence currents and voltages in that part of the system where the fault occurs, provided V_f at the fault point is chosen as the reference voltage for the calculations. However, for those parts of the system which are separated by Δ-Y transformers from the fault point, the sequence currents, and voltages calculated by bus impedance matrix must be shifted in phase before being combined to form the actual voltages. This is because the bus impedance matrices of the sequence networks are formed without consideration of phase shifts, and so they consist of per-unit impedances *referred* to the part of the network which includes the fault point.

Example 12.5. Solve for the subtransient voltages to ground at bus ②, the end of the transmission line remote from the double line-to-ground fault, in the system of Example 12.4.

Solution. Numerical values of the fault-current components are given in the solution of Example 12.4 and the elements of $\mathbf{Z}_{\text{bus}}^{(1)}$, $\mathbf{Z}_{\text{bus}}^{(2)}$, and $\mathbf{Z}_{\text{bus}}^{(0)}$ are provided in the solution of Example 12.1. Neglecting phase shift of the Δ-Y transformers for the moment and substituting the appropriate values in Eq. (12.6), we obtain for the voltages at bus ② due to the fault at bus ④,

$$V_{2a}^{(0)} = \quad -I_{fa}^{(0)}Z_{24}^{(0)} = \quad -(j1.9095)(0) \qquad = 0$$

$$V_{2a}^{(1)} = V_f - I_{fa}^{(1)}Z_{24}^{(1)} = 1 - (-j4.4342)(j0.0789) = 0.6501 \text{ per unit}$$

$$V_{2a}^{(2)} = \quad -I_{fa}^{(2)}Z_{24}^{(2)} = \quad -(-j2.5247)(j0.0789) = 0.1992 \text{ per unit}$$

Accounting for phase shift in stepping *up* to the transmission-line circuit from the fault at bus ④, we have

$$V_{2A}^{(0)} = 0$$

$$V_{2A}^{(1)} = V_{2a}^{(1)}\angle 30^\circ \quad = 0.6501\angle 30^\circ \quad = 0.5630 + j0.3251 \text{ per unit}$$

$$V_{2A}^{(2)} = V_{2a}^{(2)}\angle -30^\circ = 0.1992\angle -30^\circ = 0.1725 - j0.0996 \text{ per unit}$$

The required voltages can now be calculated:

$$V_{2A} = V_{2A}^{(0)} + V_{2A}^{(1)} + V_{2A}^{(2)} \quad = (0.5630 + j0.3251) + (0.1725 - j0.0996)$$

$$= 0.7355 + j0.2255 = 0.7693\angle 17.0^\circ \text{ per unit}$$

$$V_{2B} = V_{2A}^{(0)} + a^2V_{2A}^{(1)} + aV_{2A}^{(2)} = \left(1\angle 240^\circ\right)\left(0.6531\angle 30^\circ\right)$$

$$+\left(1\angle 120^\circ\right)\left(0.1992\angle 30^\circ\right)$$

$$= -0.1725 - j0.5535 = 0.5798\angle 107.3^\circ \text{ per unit}$$

$$V_{2C} = V_{2A}^{(0)} + aV_{2A}^{(1)} + a^2V_{2A}^{(2)} = \left(1\angle 120^\circ\right)\left(0.6531\angle 30^\circ\right)$$

$$+\left(1\angle 240^\circ\right)\left(0.1992\angle 30^\circ\right)$$

$$= -0.5656 + j0.1274 = 0.5798\angle 167.3^\circ \text{ per unit}$$

These per-unit values can be converted to volts by multiplying by the line-to-neutral base voltage $345/\sqrt{3}$ kV of the transmission line.

12.5 DEMONSTRATION PROBLEMS

Large-scale computer programs based on the bus impedance matrices of the sequence networks are generally used to analyze faults on electric utility transmission systems. Three-phase and single line-to-ground faults are usually the only types of fault studied. Since circuit-breaker applications are made according to the symmetrical short-circuit current that must be interrupted, this current is calculated for the two types of fault. The printout includes the total fault current and the contributions from each line. The results also list those quantities when each line connected to the faulted bus is opened in turn while all others are in operation.

The program uses the impedances for the lines as provided in the line data for the power-flow program and includes the appropriate reactance for each machine in forming the positive- and zero-sequence bus impedance matrices. As far as impedances are concerned, the negative-sequence network is taken to be the same as the positive-sequence network. So, for a single line-to-ground fault at bus (k), $I_{fa}^{(1)}$ is calculated in per unit as 1.0 divided by the sum $(2Z_{kk}^{(1)} + Z_{kk}^{(0)} + 3Z_f)$. The bus voltages are included in the computer printout, if called for, as well as the current in lines other than those connected to the faulted bus since this information can easily be found from the bus impedance matrices.

The following numerical examples show the analysis of a single line-to-ground fault on (1) an industrial power system and (2) a small electric utility system. Both of these systems are quite small in extent compared to the large-scale systems normally encountered. The calculations are presented without matrices in order to emphasize the circuit concepts which underlie fault analysis. The presentation should allow the reader to become more familiar with the sequence networks and how they are used to analyze faults. The principles demonstrated here are essentially the same as those employed within the large-scale computer programs used by industry. The same examples are to be solved by the bus impedance matrices in the problems at the end of this chapter.

Example 12.6. A group of identical synchronous motors is connected through a transformer to a 4.16-kV bus at a location remote from the generating plants of a power system. The motors are rated 600 V and operate at 89.5% efficiency when carrying a full load at unity power factor and rated voltage. The sum of their output ratings is 4476 kW (6000 hp). The reactances in per unit of each motor based on its own input kilovoltampere rating are $X_d'' = X_1 = 0.20$, $X_2 = 0.20$, $X_0 = 0.04$, and each is grounded through a reactance of 0.02 per unit. The motors are connected to the 4.16-kV bus through a transformer bank composed of three single-phase units, each of which is rated 2400/600 V, 2500 kVA. The 600-V windings are connected in Δ to the motors and the 2400-V windings are connected in Y. The leakage reactance of each transformer is 10%.

The power system which supplies the 4.16-kV bus is represented by a Thévenin equivalent generator rated 7500 kVA, 4.16 kV with reactances of

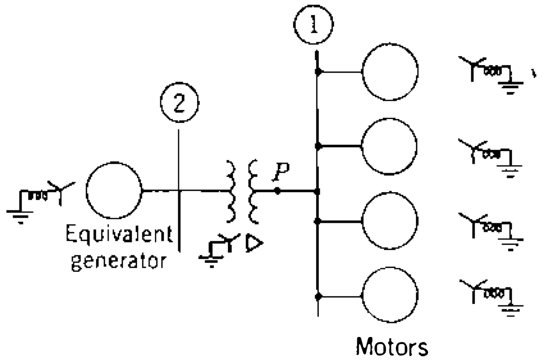


FIGURE 12.17
Single-line diagram of the system of Example 12.6.

$X_d'' = X_2 = 0.10$ per unit, $X_0 = 0.05$ per unit, and X_n from neutral to ground equal to 0.05 per unit.

Each of the identical motors is supplying an equal share of a total load of 3730 kW (5000 hp) and is operating at rated voltage, 85% power-factor lag, and 88% efficiency when a single line-to-ground fault occurs on the low-voltage side of the transformer bank. Treat the group of motors as a single equivalent motor. Draw the sequence networks showing values of the impedances. Determine the subtransient line currents in all parts of the system with prefault current neglected.

Solution. The single-line diagram of the system is shown in Fig. 12.17. The 600-V bus and the 4.16-kV bus are numbered ① and ②, respectively. Choose the rating of the equivalent generator as base: 7500 kVA, 4.16 kV at the system bus.

Since

$$\sqrt{3} \times 2400 = 4160 \text{ V} \qquad 3 \times 2500 = 7500 \text{ kVA}$$

the three-phase rating of the transformer is 7500 kVA, 4160Y/600Δ V. So, the base for the motor circuit is 7500 kVA, 600 V.

The input *rating* of the single equivalent motor is

$$\frac{6000 \times 0.746}{0.895} = 5000 \text{ kVA}$$

and the reactances of the equivalent motor in percent are the same on the base of the combined rating as the reactances of the individual motors on the base of the rating of an individual motor. The reactances of the equivalent motor in per unit on the selected base are

$$X_d'' = X_1 = X_2 = 0.2\frac{7500}{5000} = 0.3 \qquad X_0 = 0.04\frac{7500}{5000} = 0.06$$

In the zero-sequence network the reactance between neutral and ground of the equivalent motor is

$$3X_n = 3 \times 0.02\frac{7500}{5000} = 0.09 \text{ per unit}$$

and for the equivalent generator the reactance from neutral to ground is

$$3X_n = 3 \times 0.05 = 0.15 \text{ per unit}$$

Figure 12.18 shows the series connection of the sequence networks.

Since the motors are operating at rated voltage equal to the base voltage of the motor circuit, the prefault voltage of phase a at the fault bus ① is

$$V_f = 1.0 \text{ per unit}$$

Base current for the motor circuit is

$$\frac{7{,}500{,}000}{\sqrt{3} \times 600} = 7217 \text{ A}$$

and the actual motor current is

$$\frac{746 \times 5000}{0.88 \times \sqrt{3} \times 600 \times 0.85} = 4798 \text{ A}$$

Current drawn by the motor through line a before the fault occurs is

$$\frac{4798}{7217}\underline{/-\cos^{-1} 0.85} = 0.665\underline{/-31.8^\circ} = 0.565 - j0.350 \text{ per unit}$$

If prefault current is neglected, E_g'' and E_m'' are made equal to $1.0\underline{/0^\circ}$ in Fig. 12.18. Thévenin impedances are computed at bus ① in each sequence network as follows:

$$Z_{11}^{(1)} = Z_{11}^{(2)} = \frac{(j0.1 + j0.1)(j0.3)}{j(0.1 + 0.1 + 0.3)} = j0.12 \text{ per unit} \qquad Z_{11}^{(0)} = j0.15 \text{ per unit}$$

Fault current in the series connection of the sequence networks is

$$I_{fa}^{(1)} = \frac{V_f}{Z_{11}^{(1)} + Z_{11}^{(2)} + Z_{11}^{(0)}} = \frac{1.0}{j0.12 + j0.12 + j0.15} = \frac{1.0}{j0.39} = -j2.564$$

$$I_{fa}^{(2)} = I_{fa}^{(0)} = I_{fa}^{(1)} = -j2.564 \text{ per unit}$$

Current in the fault $= 3I_{fa}^{(0)} = 3(-j2.564) = -j7.692$ per unit. In the positive-sequence network the portion of $I_{fa}^{(1)}$ flowing toward P from the transformer is found by current division to be

$$\frac{-j2.564 \times j0.30}{j0.50} = -j1.538 \text{ per unit}$$

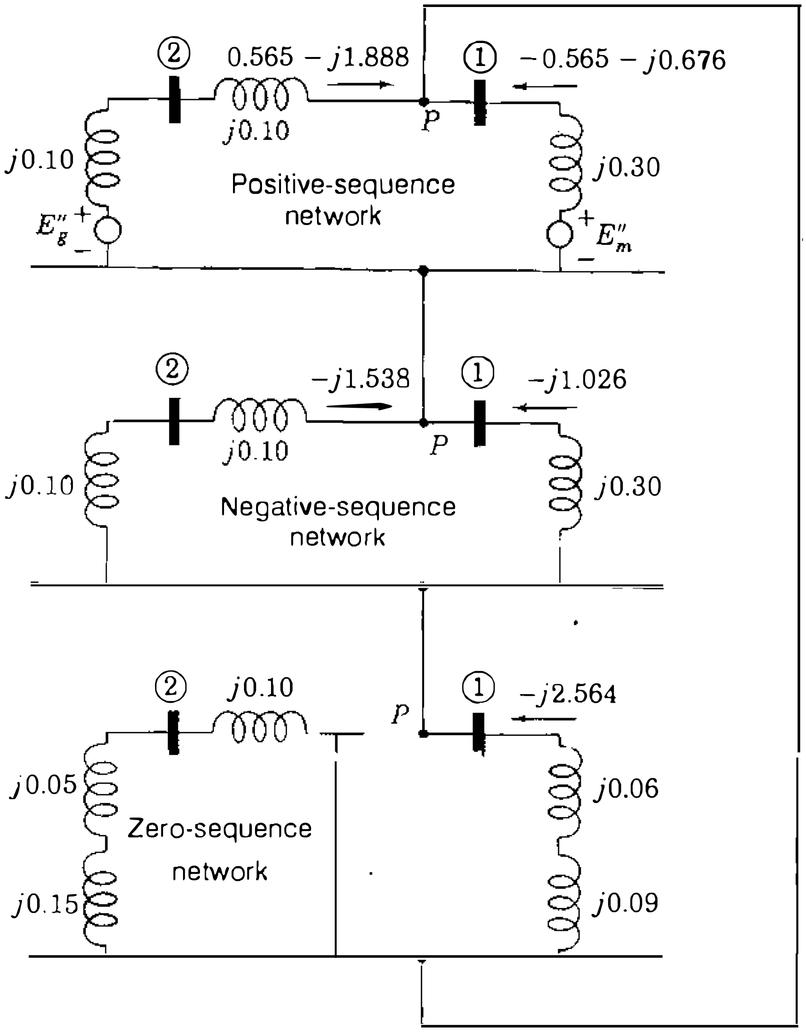


FIGURE 12.18
Connection of the sequence networks of Example 12.6. Subtransient currents are marked in per unit for a single line-to-ground fault at P. Prefault current is included.

and the portion of $I_{fa}^{(1)}$ flowing from the motor toward P is

$$\frac{-j2.564 \times j0.20}{j0.50} = -j1.026 \text{ per unit}$$

Similarly, the portion of $I_{fa}^{(2)}$ from the transformer is $-j1.538$ per unit, and the component of $I_{fa}^{(2)}$ from the motor is $-j1.026$ per unit. All of $I_{fa}^{(0)}$ flows toward P from the motor.

Currents *in the lines* at the fault, shown without subscript f, are:
To P from the transformer in per unit:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.538 \\ -j1.538 \end{bmatrix} = \begin{bmatrix} -j3.076 \\ j1.538 \\ j1.538 \end{bmatrix}$$

To P from the motors in per unit:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j2.564 \\ -j1.026 \\ -j1.026 \end{bmatrix} = \begin{bmatrix} -j4.616 \\ -j1.538 \\ -j1.538 \end{bmatrix}$$

Our method of labeling the lines is the same as in Fig. 11.23(a) such that currents $I_A^{(1)}$ and $I_A^{(2)}$ in the lines on the high-voltage side of the transformer are related to the currents $I_a^{(1)}$ and $I_a^{(2)}$ in the lines on the low-voltage side by

$$I_A^{(1)} = I_a^{(1)}\angle 30^\circ \qquad I_A^{(2)} = I_a^{(2)}\angle -30^\circ$$

Hence,

$$I_A^{(1)} = (-j1.538)\angle 30^\circ \quad = 1.538\angle -60^\circ \quad = 0.769 - j1.332$$

$$I_A^{(2)} = (-j1.538)\angle -30^\circ \; = 1.538\angle -120^\circ \; = -0.769 - j1.332$$

and from Fig. 12.18 we note that $I_A^{(0)} = 0$ in the zero-sequence network. Since there are no zero-sequence currents on the high-voltage side of the transformer, we have

$$I_A \;= I_A^{(1)} + I_A^{(2)} = (0.769 - j1.332) + (-0.769 - j1.332) = -j2.664 \text{ per unit}$$

$$I_B^{(1)} = a^2 I_A^{(1)} = (1\angle 240^\circ)(1.538\angle -60^\circ) \; = -1.538 + j0$$

$$I_B^{(2)} = a\ I_A^{(2)} = (1\angle 120^\circ)(1.538\angle -120^\circ) = \quad 1.538 + j0$$

$$I_B \;= I_B^{(1)} + I_B^{(2)} = 0$$

$$I_C^{(1)} = a\ I_A^{(1)} = (1\angle 120^\circ)(1.538\angle -60^\circ) \; = \quad 0.769 + j1.332$$

$$I_C^{(2)} = a^2 I_A^{(2)} = (1\angle 240^\circ)(1.538\angle -120^\circ) = -0.769 + j1.332$$

$$I_C \;= I_C^{(1)} + I_C^{(2)} = j2.664 \text{ per unit}$$

If voltages throughout the system are to be found by circuit analysis, their components at any point can be calculated from the currents and reactances of the sequence networks. Components of voltages on the high-voltage side of the transformer are found first without regard for phase shift. Then, the effect of phase shift must be determined.

By evaluating the base currents on the two sides of the transformer, we can convert the above per-unit currents to amperes. Base current for the motor circuit was found previously and equals 7217 A. Base current for the high-voltage circuit is

$$\frac{7{,}500{,}000}{\sqrt{3} \times 4160} = 1041 \text{ A}$$

Current in the fault is

$$7.692 \times 7217 = 55{,}500 \text{ A}$$

Currents in the lines between the transformer and the fault are

In line a: $3.076 \times 7217 = 22{,}200$ A

In line b: $1.538 \times 7217 = 11{,}100$ A

In line c: $1.538 \times 7217 = 11{,}100$ A

Currents in the lines between the motor and the fault are

In line a: $4.616 \times 7271 = 33{,}300$ A

In line b: $1.538 \times 7217 = 11{,}100$ A

In line c: $1.538 \times 7217 = 11{,}100$ A

Currents in the lines between the 4.16 kV bus and the transformer are

In line A: $2.664 \times 1041 = 2773$ A

In line B: 0

In line C: $2.664 \times 1041 = 2773$ A

The currents we have calculated in the above example are those which would flow upon the occurrence of a single line-to-ground fault when there is no load on the motors. These currents are correct only if the motors are drawing no current whatsoever. The statement of the problem specifies the load conditions at the time of the fault, however, and the load can be considered. To account for the load, we add the per-unit current drawn by the motor through line a before the fault occurs to the portion of $I_{fa}^{(1)}$ flowing toward P from the transformer and subtract the same current from the portion of $I_{fa}^{(1)}$ flowing from the motor to P. The new value of positive-sequence current from the transformer to the fault in phase a is

$$0.565 - j0.350 - j1.538 = 0.565 - j1.888$$

and the new value of positive-sequence current from the motor to the fault in

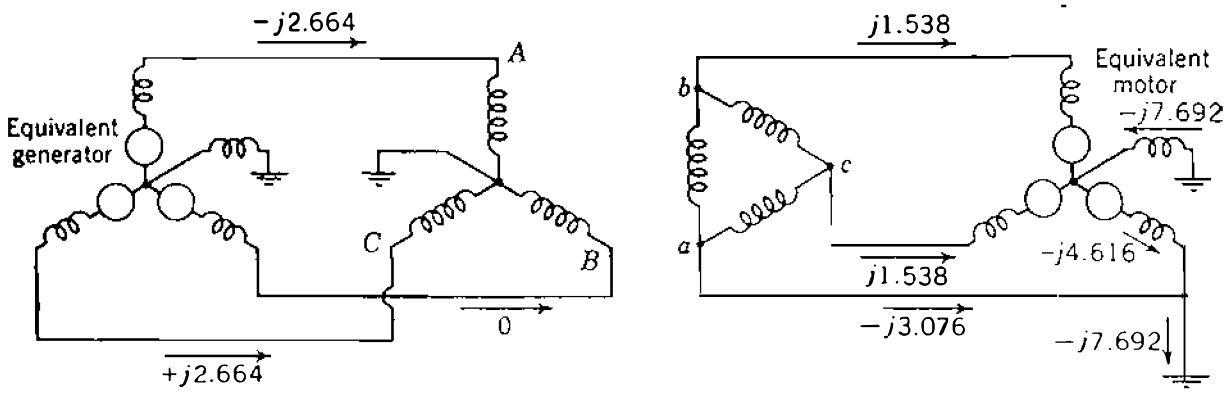


FIGURE 12.19
Per-unit values of subtransient line currents in all parts of the system of Example 12.6, prefault current neglected.

phase a is

$$-0.565 + j0.350 - j1.026 = -0.565 - j0.676$$

These values are shown in Fig. 12.18. The remainder of the calcula these new values, proceeds as in the example.

Figure 12.19 gives the per-unit values of subtransient line currents in all parts of the system when the fault occurs at no load. Figure 12.20 shows the values for the fault occurring on the system when the load specified in the example is considered. In a larger system where the fault current is much higher in comparison with the load current the effect of neglecting the load current is less than is indicated by comparing Figs. 12.19 and 12.20. In the la however, the prefault currents determined by a power-flow study could simply be added to the fault current found with the load neglected.

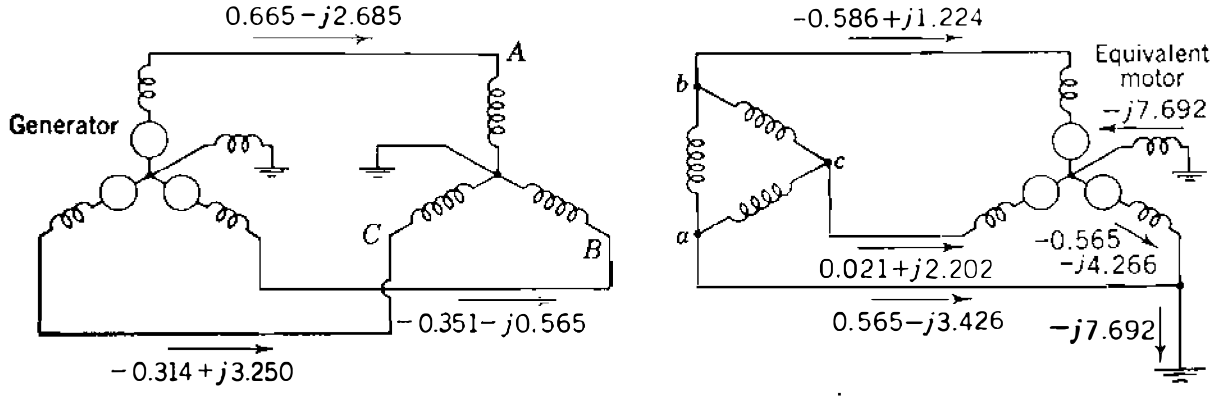


FIGURE 12.20
Per-unit values of subtransient line currents in all parts of the system of Example 12.6, prefault current considered.

Example 12.7. The single-line diagram of a small power system is shown **in Fig.** 12.21. A bolted single line-to-ground fault at point P is to be analyzed. The **ratings** and reactances of the generator and the transformers are

$$\text{Generator:}\quad 100\ \text{MVA}, 20\ \text{kV};\quad X'' = X_2 = 20\%,\quad X_0 = 4\%,\quad X_n = 5\%$$

$$\text{Transformers } T_1 \text{ and } T_2\text{:}\quad 100\ \text{MVA}, 20\Delta/345\text{Y kV};\quad X = 10\%$$

On a chosen base of 100 MVA, 345 kV in the transmission-line circuit the line reactances are

$$\text{From } T_1 \text{ to } P\text{:}\quad X_1 = X_2 = 20\%,\quad X_0 = 50\%$$

$$\text{From } T_2 \text{ to } P\text{:}\quad X_1 = X_2 = 10\%,\quad X_0 = 30\%$$

To simulate the fault, the sequence networks of the system with reactances marked in per unit are connected in series, as shown in Fig. 12.22. Verify the values of the currents shown in the figure and draw a complete three-phase circuit diagram with all current flows marked in per unit. Assume that the transformers are lettered so that Eqs. (11.88) apply.

Solution. With switch S open, the prefault currents are zero and the open-circuit voltage of phase A at point P can be taken as the reference voltage $1.0 + j0.0$ per unit. The impedances seen looking into the sequence networks at the fault point are

$$Z_{pp}^{(0)} = \frac{(j0.6)(j0.4)}{j0.6 + j0.4} = j0.24 \text{ per unit}$$

$$Z_{pp}^{(1)} = Z_{pp}^{(2)} = j0.5 \text{ per unit}$$

The sequence currents in the hypothetical stub of phase A at P are

$$I_{fA}^{(0)} = I_{fA}^{(1)} = I_{fA}^{(2)} = \frac{1.0 + j0.0}{j0.5 + j0.5 + j0.24} = -j0.8065 \text{ per unit}$$

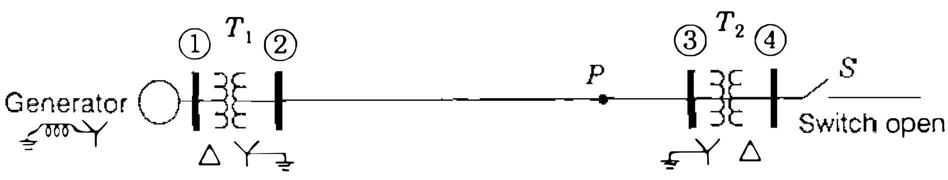


FIGURE 12.21
Single-line diagram of the system of Example 12.7. Single line-to-ground fault is at point P.

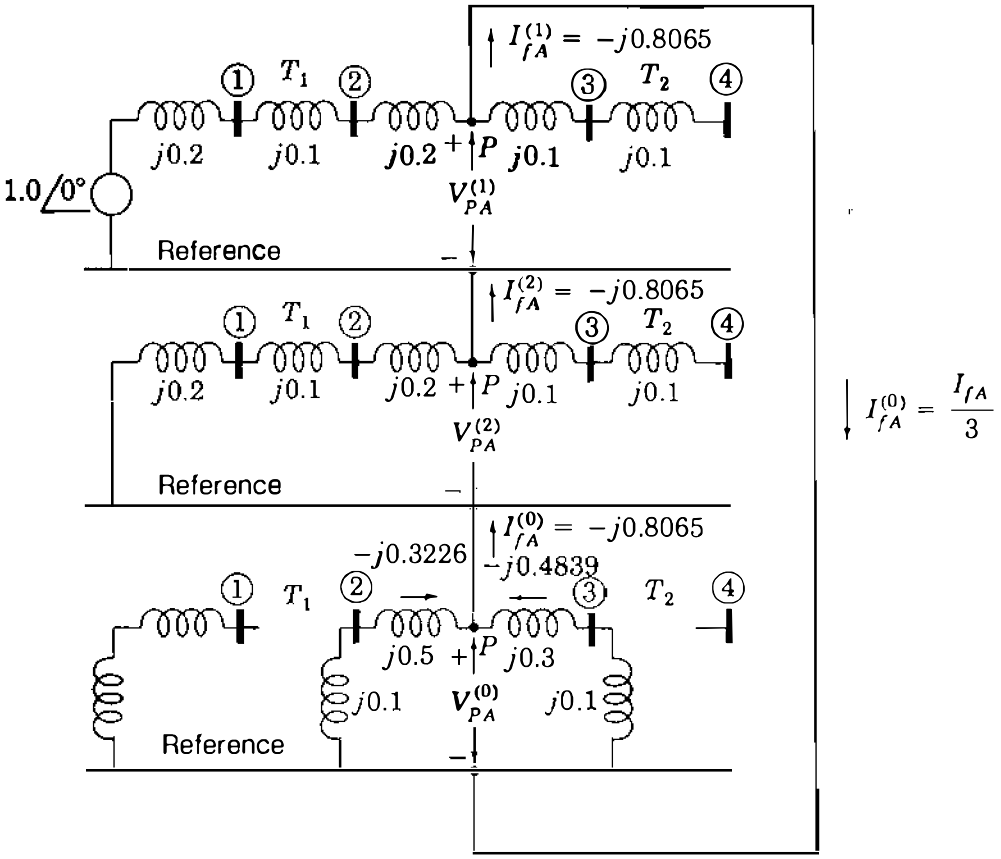


FIGURE 12.22
Connection of the sequence networks for the system of Fig. 12.21 to simulate single line-to-ground fault at point P.

The total current in the fault is

$$I_{fA} = 3I_{fA}^{(0)} = -j2.4195 \text{ per unit}$$

In the stub of phase B at point P we have

$$I_{fB}^{(1)} = a^2 I_{fA}^{(1)} = 0.8065\angle -90° + 240° = 0.8065\angle 150°$$

$$I_{fB}^{(2)} = aI_{fA}^{(2)} = 0.8065\angle -90° + 120° = 0.8065\angle 30°$$

$$I_{fB}^{(0)} = I_{fA}^{(0)} \qquad = 0.8065\angle -90°$$

$$I_{fB} = I_{fB}^{(0)} + I_{fB}^{(1)} + I_{fB}^{(2)} = 0$$

Likewise, in the stub of phase C at point P we have

$$I_{fC} = I_{fC}^{(0)} + I_{fC}^{(1)} + I_{fC}^{(2)} = 0$$

In the zero-sequence network the currents are:

Toward P from T_1

$$I_A^{(0)} = \frac{j0.4}{j0.6 + j0.4}\left(0.8065\angle -90^\circ\right) = 0.3226\angle -90^\circ \text{ per unit}$$

Toward P from T_2

$$I_A^{(0)} = \frac{j0.6}{j0.6 + j0.4}\left(0.8065\angle -90^\circ\right) = 0.4839\angle -90^\circ \text{ per unit}$$

In the transmission line the currents are:

Toward P from T_1

In line A: $0.3226\angle -90^\circ + 0.8065\angle -90^\circ + 0.8065\angle -90^\circ = -j1.9356$ per unit

In line B: $0.3226\angle -90^\circ + 0.8065\angle 150^\circ + 0.8065\angle 30^\circ = j0.4839$ per unit

In line C: $0.3226\angle -90^\circ + 0.8065\angle 30^\circ + 0.8065\angle 150^\circ = j0.4839$ per unit

Toward P from T_2

In line A: $I_A = -j0.4839$ per unit

In line B: $I_B = -j0.4839$ per unit

In line C: $I_C = -j0.4839$ per unit

Note that positive-, negative-, and zero-sequence components of current flow in lines A, B, and C from T_1 but only zero-sequence components flow in these lines from T_2. Kirchhoff's current law is fulfilled, however.
In the generator the currents are

$$I_a = I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = 0 + 0.8065\angle -90^\circ - 30^\circ + 0.8065\angle -90^\circ + 30^\circ$$
$$= -j1.3969$$

$$I_b = I_a^{(0)} + a^2 I_a^{(1)} + a\, I_a^{(2)} = 0 + 0.8065\angle -120^\circ + 240^\circ + 0.8065\angle -60^\circ + 120^\circ$$
$$= j1.3969$$

$$I_c = I_a^{(0)} + a\, I_a^{(1)} + a^2 I_a^{(2)} = 0 + 0.8065\angle -120^\circ + 120^\circ + 0.8065\angle -60^\circ + 240^\circ$$
$$= 0$$

The three-phase circuit diagram of Fig. 12.23 shows all the current flows in per

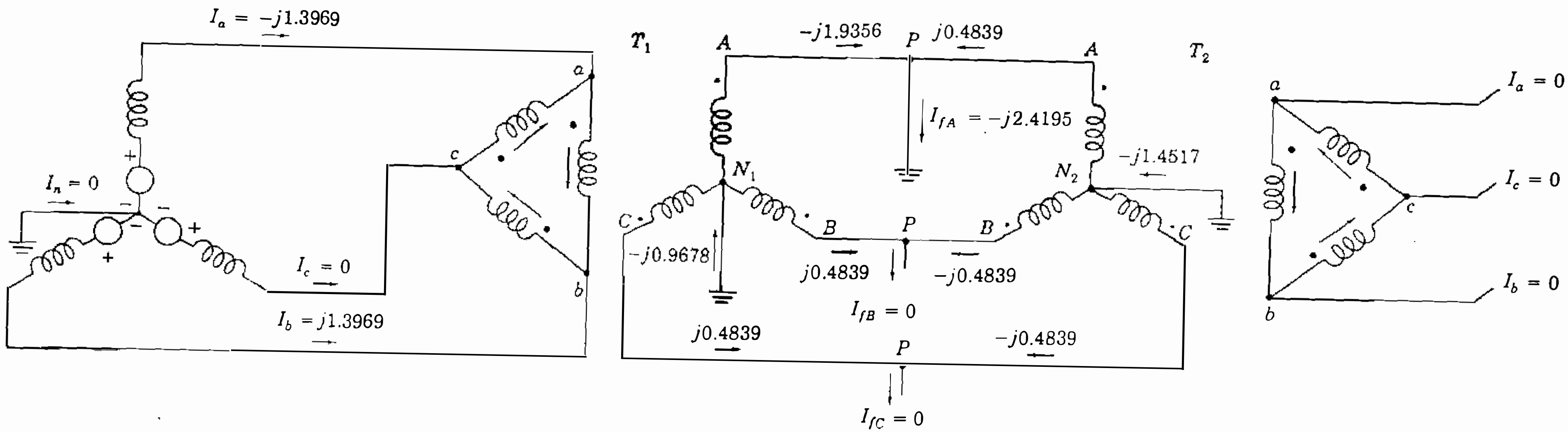


FIGURE 12.23
Current flows in

the system of Fi

unit. From this diagram note that:

- Lines are lettered and polarity marks are placed so that Eqs. (11.88) are valid.
- Stubs are connected to each line at the fault.
- For a single line-to-ground fault stub currents $I_B = I_C = 0$, but $I_B^{(0)}$, $I_B^{(1)}$, $I_B^{(2)}$, $I_C^{(0)}$, $I_C^{(1)}$, and $I_C^{(2)}$ in the stubs all have nonzero values.
- Fault current flows out of stub A, then partly to T_1 and partly to T_2.
- In the generator only positive- and negative-sequence currents are flowing.
- In the Δ windings of T_2 only zero-sequence currents are flowing.
- In the Δ windings of T_1 each phase winding contains positive-, negative-, and zero-sequence components of current. These components are shown in Fig.

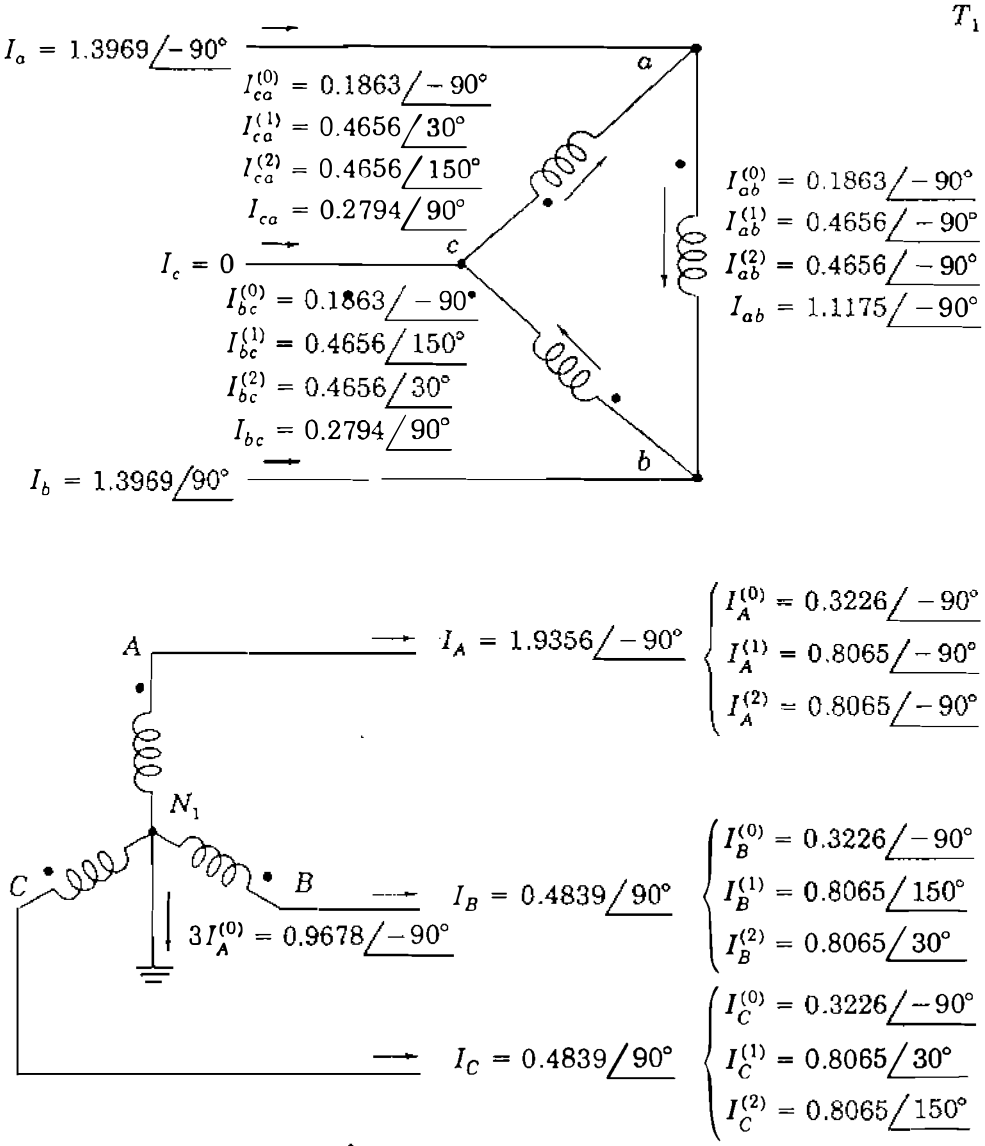


FIGURE 12.24
Symmetrical components of currents in transformer T_1 of Fig. 12.23.

12.24 and yield

$$I_{ab} = \frac{I_A}{\sqrt{3}} = 1.1175\underline{/-90^\circ}$$

$$I_{bc} = \frac{I_B}{\sqrt{3}} = 0.2794\underline{/90^\circ}$$

$$I_{ca} = \frac{I_C}{\sqrt{3}} = 0.2794\underline{/90^\circ}$$

12.6 OPEN-CONDUCTOR FAULTS

When one phase of a bal
created and asymmetrical currents flow. A simil
when any two of the three phases are opened while the third phase remains closed. These unbalanced conditions are caused, for example, when one- or two-phase conductors of a transmission line are physical
storm. In other circuits, due to current overload, fuses or other switching devices may operate in one or two conductors and fail to
conductors. Such open-conductor faults can be analyzed by means of the bus impedance matrices of the sequence networks, as we now demonstrate.

Figure 12.25 depicts a section of a three-phase circuit in which the line currents in the respective phases are I_a, I_b, and I_c, with positi
bus (m) to bus (n) as shown. Phase a is open between points p and p' in Fig. 12.25(a), whereas phases b and c are open between the same two points in Fig. 12.25(b). The same open-conductor fault
phases are first opened between points p and p', and *short circuits* are then applied in those phases which are shown to be closed in Fig. 12.25. The ensuing development follows this reasoning.

Opening the three phases is the same as removing line (m) - (n) altogether and then adding appropriate impedances from buses (m) and (n) to the points p and p'. If li
simulate the opening of the three phases by adding the negative impedances $-Z_0$, $-Z_1$, and $-Z_2$ between buses (m) and (n) in the corresponding Thévenin equivalents of the three sequence networks of the *intact* system. To exemplify, consider Fig. 12.26(a), which shows the connection of $-Z_1$ to
sequence Thévenin equivalent between buses (m) and (n). The impedances shown are the elements $Z_{mm}^{(1)}$, $Z_{nn}^{(1)}$, and $Z_{mn}^{(1)} = Z_{nm}^{(1)}$ of the positive-sequence bus impedance matrix $\mathbf{Z}_{\text{bus}}^{(1)}$ of the intact system, and $Z_{\text{th},mn}^{(1)} = Z_{mm}^{(1)} + Z_{nm}^{(1)} - 2Z_{mn}^{(1)}$ is the corresponding Thévenin impedance between buses (m) and (n). Voltages V_m and V_n are the normal (positive-sequence) voltages of phase a at buses (m) and (n) before the open-conductor faults occur. The positive-sequence impedances kZ_1 and $(1-k)Z_1$, where $0 \le k \le 1$, are added as shown to

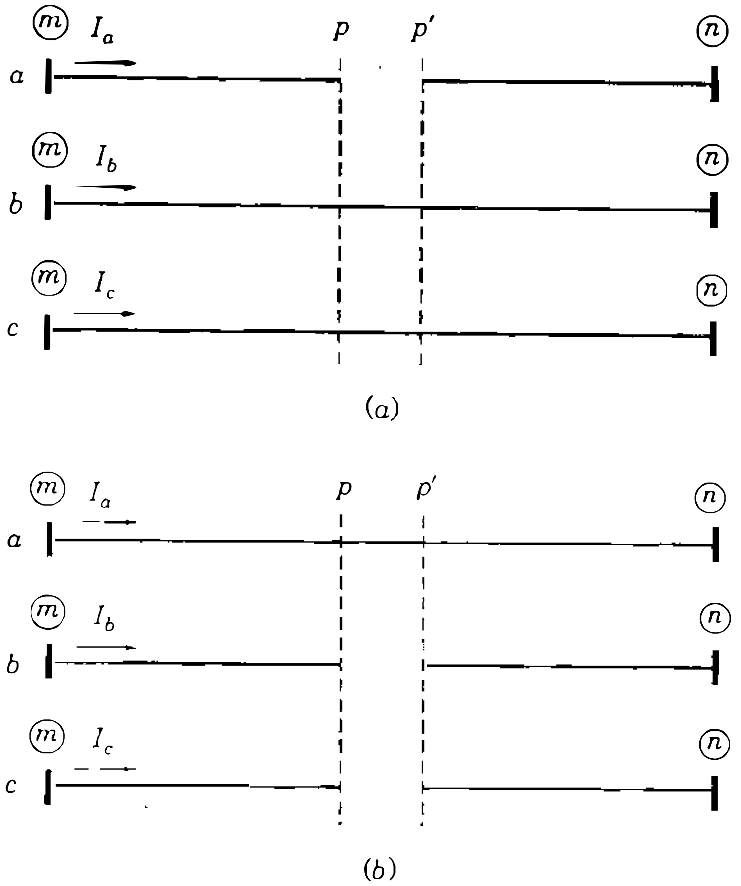


FIGURE 12.25
Open-conductor faults on a section of a three-phase system between buses (m) and (n): (*a*) conductor *a* open; (*b*) conductors *b* and *c* open between points *p* and *p*′.

represent the fractional lengths of the broken line (m)- (n) from bus (m) to point *p* and bus (n) to point *p*′, respectively. To use a convenient notation, let the voltage $V_a^{(1)}$ denote the phase-*a* positive-sequence component of the *voltage drops* $V_{pp',a}$, $V_{pp',b}$, and $V_{pp',c}$ from *p* to *p*′ in the phase conductors. We shall soon see that $V_a^{(1)}$, and the corresponding negative- and zero-sequence components $V_a^{(2)}$ and $V_a^{(0)}$, take on different values depending on which one of the open-conductor fault

By source transformation we can replace the voltage drop $V_a^{(1)}$ in series with the impedance $[kZ_1 + (1 - k)Z_1]$ in Fi
in parallel with the impedance Z_1, as shown in Fig. 12.26(*b*). In this latter figure the parallel combination of $-Z_1$ and Z_1 can be canceled, as shown in Fig. 12.26(*c*).

The above considerations for the positive-sequence network also apply directly to the negative- and zero-sequence networks, but we must remember that the latter networks do not contain any internal sources of their own. In drawi
understood that the currents $V_a^{(2)}/Z_2$ and $V_a^{(0)}/Z_0$, like the current $V_a^{(1)}/Z_1$ of Fig. 12.26(*c*), owe their origin to the open-conductor fault between points *p* and *p*′ in the system. If there is no open conductor, the voltages $V_a^{(1)}$, $V_a^{(2)}$, and $V_a^{(0)}$ are al
each of the sequence currents $V_a^{(0)}/Z_0$, $V_a^{(1)}/Z_1$, and $V_a^{(2)}/Z_2$ can be regarded in turn as a *pair* of injections into buses (m) and (n) of the corresponding

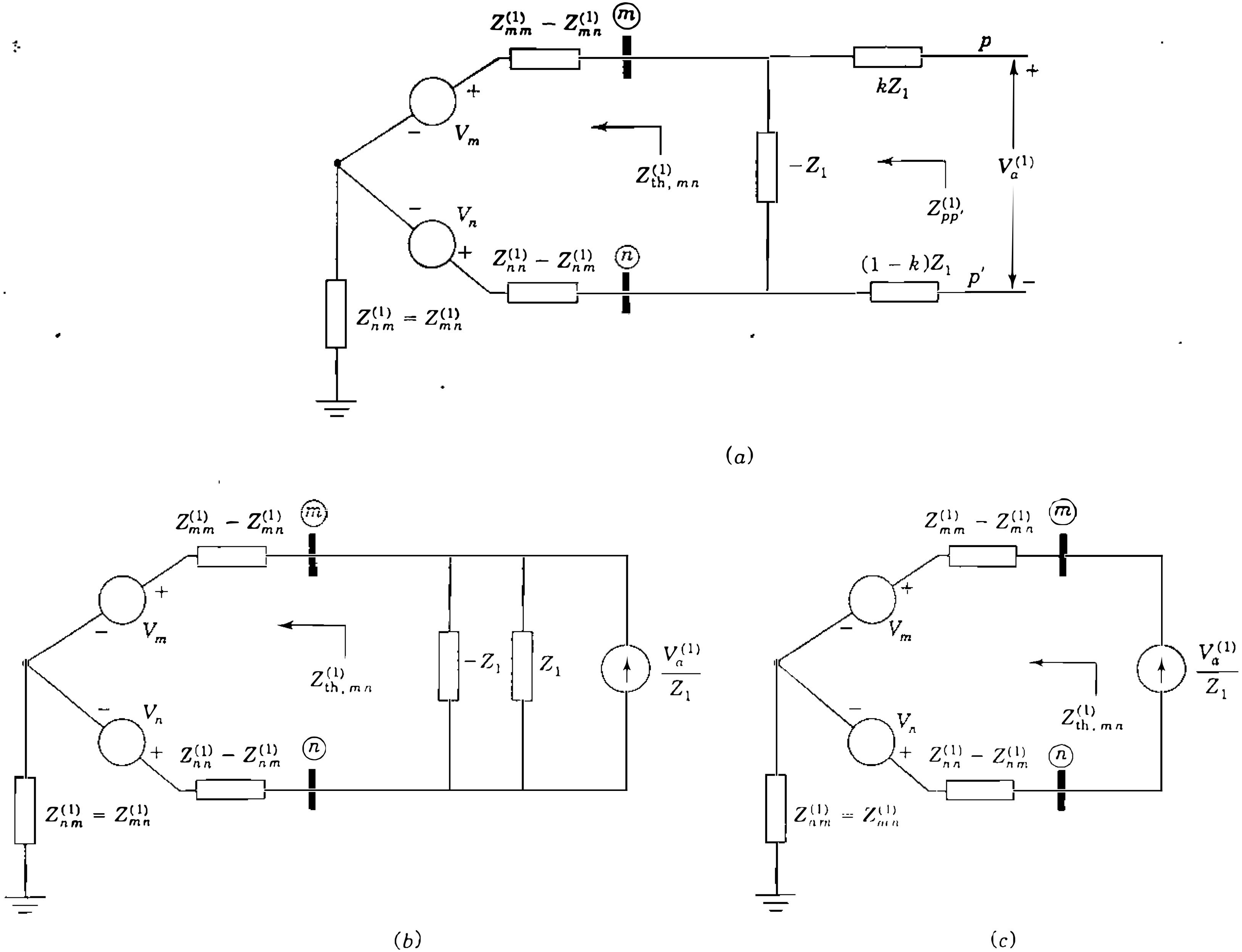


FIGURE 12.26
Simulating the opening of line
svstem; (b) transformation to current source; (c) resultant equivalent circuit.

sequence network of the *intact* system. Hence, we can use the bus impedance matrices $\mathbf{Z}_{\text{bus}}^{(0)}$, $\mathbf{Z}_{\text{bus}}^{(1)}$, and $\mathbf{Z}_{\text{bus}}^{(2)}$ of the *normal* configuration of the system to determine the voltage changes due to the open-conductor faults. But first we must find expressions for the symmetrical components $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$ of the voltage drops across the fault points p and p' for each type of fault shown in Fig. 12.25. These voltage drops can be regarded as giving rise to the following sets of injection currents into the sequence networks of the normal system configuration:

	Positive Sequence	Negative Sequence	Zero Sequence
At bus Ⓜ:	$\dfrac{V_a^{(1)}}{Z_1}$	$\dfrac{V_a^{(2)}}{Z_2}$	$\dfrac{V_a^{(0)}}{Z_0}$
At bus Ⓝ:	$-\dfrac{V_a^{(1)}}{Z_1}$	$-\dfrac{V_a^{(2)}}{Z_2}$	$-\dfrac{V_a^{(0)}}{Z_0}$

as shown in Figs. 12.26 and 12.27. By multiplying the bus impedance matrices $\mathbf{Z}_{\text{bus}}^{(0)}$, $\mathbf{Z}_{\text{bus}}^{(1)}$, and $\mathbf{Z}_{\text{bus}}^{(2)}$ by current vectors containing only these current injections, we obtain the following *changes* in the symmetrical components of the phase-a voltage of each bus ⓘ:

$$\text{Zero sequence:} \quad \Delta V_i^{(0)} = \frac{Z_{im}^{(0)} - Z_{in}^{(0)}}{Z_0} V_a^{(0)}$$

$$\text{Positive sequence:} \quad \Delta V_i^{(1)} = \frac{Z_{im}^{(1)} - Z_{in}^{(1)}}{Z_1} V_a^{(1)} \tag{12.27}$$

$$\text{Negative sequence:} \quad \Delta V_i^{(2)} = \frac{Z_{im}^{(2)} - Z_{in}^{(2)}}{Z_2} V_a^{(2)}$$

Before developing the equations for $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$ for each type of open-conductor fault, let us derive expressions for the Thévenin equivalent impedances of the sequence networks, as seen from fault points p and p'.

Looking into the positive-sequence network of Fig. 12.26(a) between p and p', we see the impedance $Z_{pp'}^{(1)}$ given by

$$Z_{pp'}^{(1)} = kZ_1 + \frac{Z_{\text{th},mn}^{(1)}(-Z_1)}{Z_{\text{th},mn}^{(1)} - Z_1} + (1-k)Z_1 = \frac{-Z_1^2}{Z_{\text{th},mn}^{(1)} - Z_1} \tag{12.28}$$

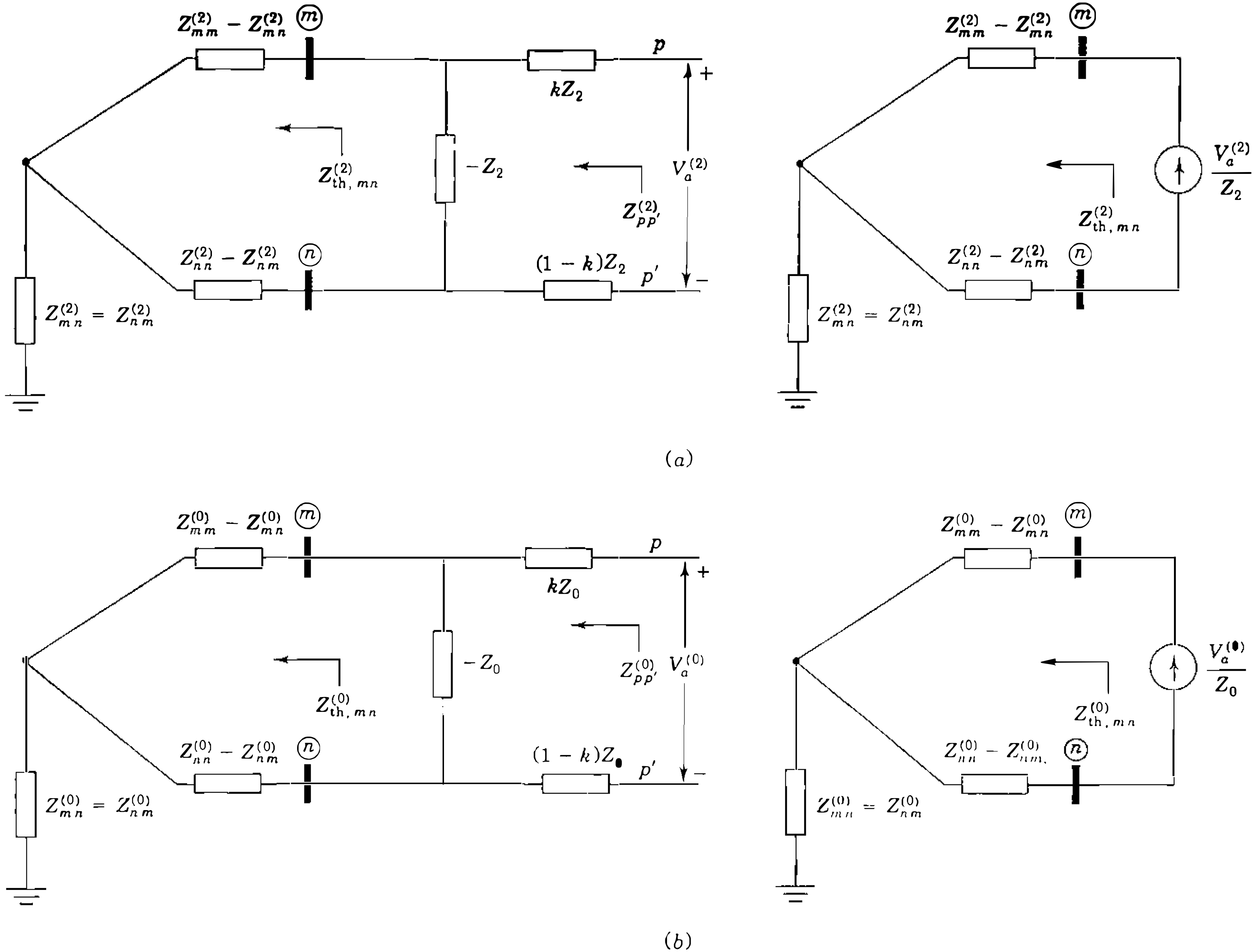


FIGURE 12.27
Simulating the opening of line (m)–(n) between points p and p': (a) negative-sequence and (b) zero-sequence equivalent circuits.

and from p to p' the open-circuit voltage obtained by voltage division is

$$\text{Open-circuit voltage from } p \text{ to } p' = \frac{-Z_1}{Z_{\text{th},mn}^{(1)} - Z_1}(V_m - V_n) = \frac{Z_{pp'}^{(1)}}{Z_1}(V_m - V_n) \tag{12.29}$$

Before any conductor opens, the current I_{mn} in phase a of the line ⓜ – ⓝ is positive sequence and is given by

$$I_{mn} = \frac{V_m - V_n}{Z_1} \tag{12.30}$$

Substituting this expression for I_{mn} in Eq. (12.29), we obtain

$$\text{Open-circuit voltage from } p \text{ to } p' = I_{mn} Z_{pp'}^{(1)} \tag{12.31}$$

Figure 12.28(a) shows the resulting positive-sequence equivalent circuit between points p and p'. Analogous to Eq. (12.28) we have

$$Z_{pp'}^{(2)} = \frac{-Z_2^2}{Z_{\text{th},mn}^{(2)} - Z_2} \qquad \text{and} \qquad Z_{pp'}^{(0)} = \frac{-Z_0^2}{Z_{\text{th},mn}^{(0)} - Z_0} \tag{12.32}$$

which are the negative- and zero-sequence impedances, respectively, between p and p' in Figs. 12.28(b) and 12.28(c). We can now proceed to develop expressions for the sequence voltage drops $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$.

One open conductor

Let us consider one open conductor as in Fig. 12.25(a). Owing to the open circuit in phase a, the current $I_a = 0$, and so

$$I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = 0 \tag{12.33}$$

where $I_a^{(0)}$, $I_a^{(1)}$, and $I_a^{(2)}$ are the symmetrical components of the *line* currents I_a, I_b, and I_c from p to p'. Since phases b and c are closed, we also have the voltage drops

$$V_{pp',b} = 0 \qquad V_{pp',c} = 0 \tag{12.34}$$

Resolving the series voltage drops across the fault point into their symmetrical

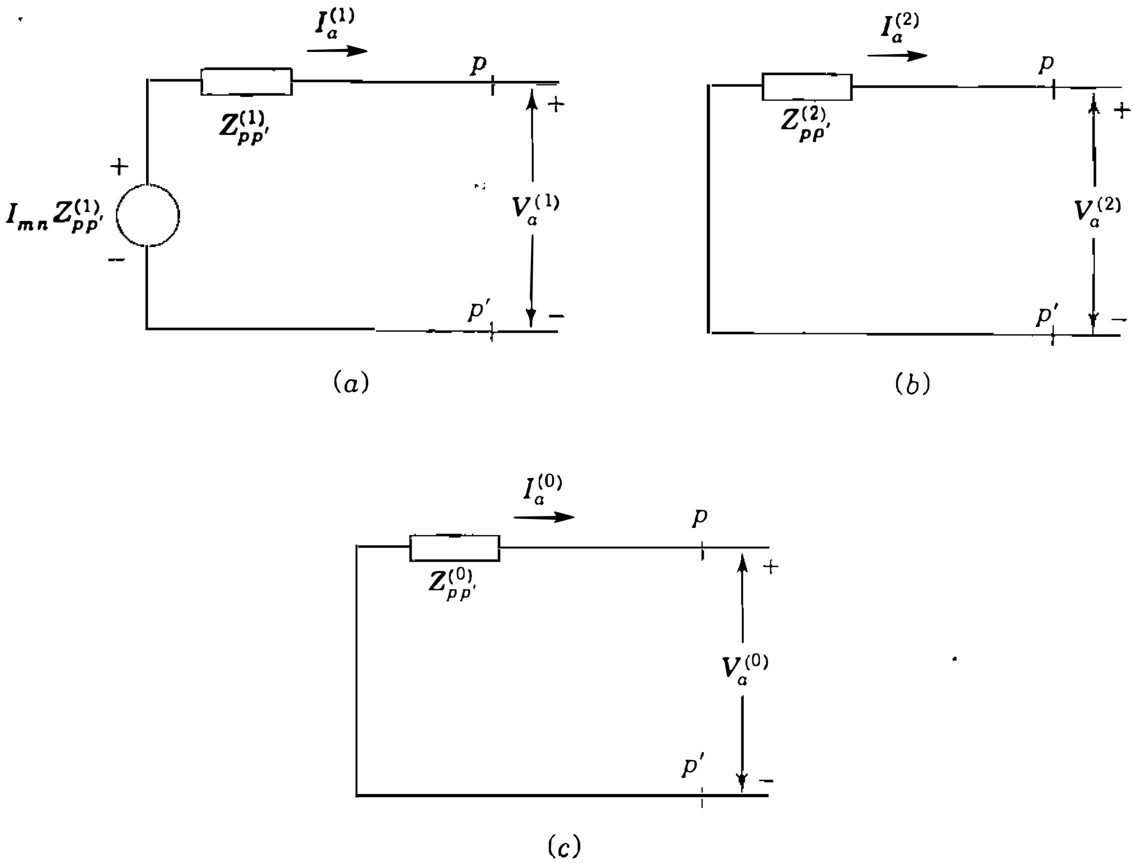


FIGURE 12.28
Looking into the system between points p and p': (a) positive-sequence, (b) negative-sequence, and (c) zero-sequence equivalent circuits.

components, we obtain

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{pp',a} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_{pp',a} \\ V_{pp',a} \\ V_{pp',a} \end{bmatrix} \tag{12.35}$$

That is,

$$V_a^{(0)} = V_a^{(1)} = V_a^{(2)} = \frac{V_{pp',a}}{3} \tag{12.36}$$

In words, this equation states that the open conductor in phase a causes equal voltage drops to appear from p to p' in each of the sequence networks. We can satisfy this requirement and that of Eq. (12.33) by connecting the Thévenin equivalents of the sequence networks *in parallel* at the points p and p', as shown in Fig. 12.29. From this figure the expression for the positive-sequence

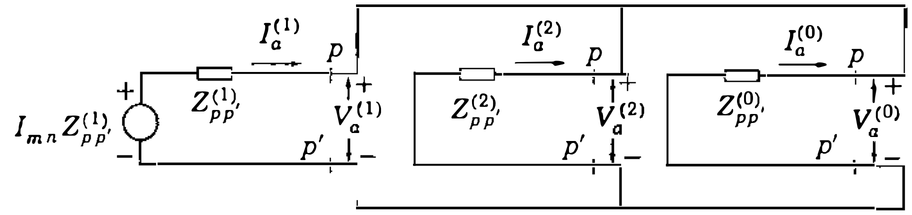


FIGURE 12.29
Connection of the sequence networks of the
and p'.

current $I_a^{(1)}$ is found to be

$$I_a^{(1)} = I_{mn} \frac{Z_{pp'}^{(1)}}{Z_{pp'}^{(1)} + \dfrac{Z_{pp'}^{(2)} Z_{pp'}^{(0)}}{Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}}$$

$$= I_{mn} \frac{Z_{pp'}^{(1)}\left(Z_{pp'}^{(2)} + Z_{pp'}^{(0)}\right)}{Z_{pp'}^{(0)} Z_{pp'}^{(1)} + Z_{pp'}^{(1)} Z_{pp'}^{(2)} + Z_{pp'}^{(2)} Z_{pp'}^{(0)}} \tag{12.37}$$

The sequence voltage drops $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$ are then given by Fig. 12.29 as

$$V_a^{(0)} = V_a^{(2)} = V_a^{(1)} = I_a^{(1)} \frac{Z_{pp'}^{(2)} Z_{pp'}^{(0)}}{Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$$

$$= I_{mn} \frac{Z_{pp'}^{(0)} Z_{pp'}^{(1)} Z_{pp'}^{(2)}}{Z_{pp'}^{(0)} Z_{pp'}^{(1)} + Z_{pp'}^{(1)} Z_{pp'}^{(2)} + Z_{pp'}^{(2)} Z_{pp'}^{(0)}} \tag{12.38}$$

The quantities on the right-hand side of this equation are known from the impedance parameters of the sequence networks and the prefault current in phase a of the line (m)– (n). Thus, the currents $V_a^{(0)}/Z_0$, $V_a^{(1)}/Z_1$, and $V_a^{(2)}/Z_2$ for injection into the corresponding sequence networks can be determined from Eq. (12.38).

Two open conductors

When two conductors are open, as shown in Fig. 12.25(b), we have fault conditions which are the *duals*[2] of those in Eqs. (12.33) and (12.34); namely,

$$V_{pp',a} = V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = 0 \tag{12.39}$$

$$I_b = 0 \qquad I_c = 0 \tag{12.40}$$

[2] *Duality* is treated in many textbooks

Resolving the line currents into their symmetrical components gives

$$I_a^{(0)} = I_a^{(1)} = I_a^{(2)} = \frac{I_a}{3} \tag{12.41}$$

Equations (12.39) and (12.41) are both satisfied by connecting the Thévenin equivalent of the negative- and zero-sequence networks *in series* between points p and p', as shown in Fig. 12.30. The sequence currents are now expressed by

$$I_a^{(0)} = I_a^{(2)} = I_a^{(1)} = I_{mn}\frac{Z_{pp'}^{(1)}}{Z_{pp'}^{(0)} + Z_{pp'}^{(1)} + Z_{pp'}^{(2)}} \tag{12.42}$$

where I_{mn} is again the prefault current in phase a of the line ⓜ- ⓝ before the open circuits occur in phases b and c. The sequence voltage drops are now given by

$$\begin{aligned}
V_a^{(1)} &= I_a^{(1)}\left(Z_{pp'}^{(2)} + Z_{pp'}^{(0)}\right) = I_{mn}\frac{Z_{pp'}^{(1)}\left(Z_{pp'}^{(2)} + Z_{pp'}^{(0)}\right)}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)} + Z_{pp'}^{(0)}} \\
V_a^{(2)} &= -I_a^{(2)}Z_{pp'}^{(2)} \qquad\qquad = I_{mn}\frac{-Z_{pp'}^{(1)}Z_{pp'}^{(2)}}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)} + Z_{pp'}^{(0)}} \\
V_a^{(0)} &= -I_a^{(0)}Z_{pp'}^{(0)} \qquad\qquad = I_{mn}\frac{-Z_{pp'}^{(1)}Z_{pp'}^{(0)}}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}
\end{aligned} \tag{12.43}$$

In each of these equations the right-hand side quantities are all known before the fault occurs. Therefore, Eq. (12.38) can be used to evaluate the symmetrical components of the voltage drops between the fault points p and p' when an open-conductor fault occurs; and Eq. (12.43) can be similarly used when a fault due to two open conductors occurs.

The net effect of the open conductors on the positive-sequence network is to increase the *transfer impedance* across the line in which the open-conductor fault occurs. For one open conductor this *increase* in impedance equals the *parallel* combination of the negative- and zero-sequence networks between points p and p'; for two open conductors the *increase* in impedance equals the *series* combination of the negative- and zero-sequence networks between points p and p'.

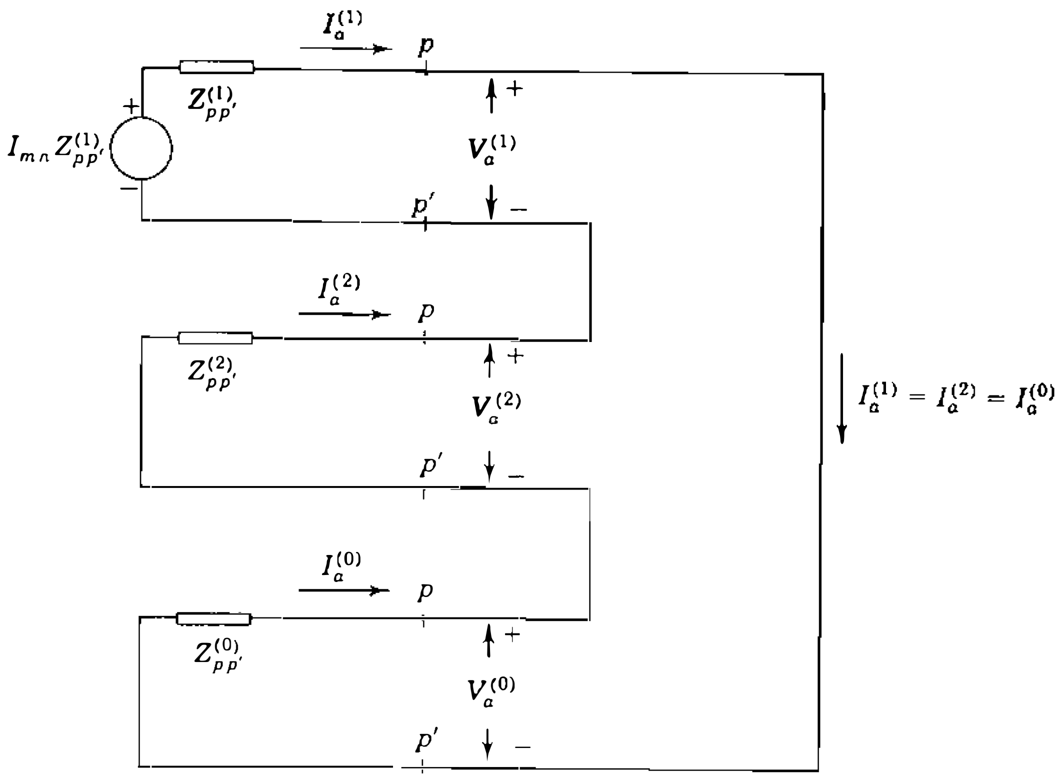


FIGURE 12.30
Connection of the points p and p'.

Example 12.8. In the system of Fig. 12.5 consider that machine 2 is a motor drawing a load equivalent to 50 MVA at 0.8 power-factor lagging and nominal system voltage of 345 kV at bus ③. Determine the change in voltage at bus ③ when the transmission line undergoes (*a*) a one-open-conductor fault and (*b*) a two-open-conductor fault along its span between buses ② and ③. Choose a base of 100 MVA, 345 kV in the transmission line.

Solution. All t
choosing the
current in line [redacted]- ③ as follows:

$$I_{23} = \frac{P - jQ}{V_3^*} = \frac{0.5(0.8 - j0.6)}{1.0 + j0.0} = 0.4 - j0.3 \text{ per unit}$$

The sequence networks of Fig. 12.6 show that line ②- ③ has parameters

$$Z_1 = Z_2 = j0.15 \text{ per unit} \qquad Z_0 = j0.50 \text{ per unit}$$

The bus impedance matrices $\mathbf{Z}_{\text{bus}}^{(0)}$ and $\mathbf{Z}_{\text{bus}}^{(1)} = \mathbf{Z}_{\text{bus}}^{(2)}$ are also given in Example 12.1. Designating the open-circuit points of the line as p and p', we can calculate from

Eqs. (12.28) and (12.32)

$$Z_{pp'}^{(1)} = Z_{pp'}^{(2)} = \frac{-Z_1^2}{Z_{22}^{(1)} + Z_{33}^{(1)} - 2Z_{23}^{(1)} - Z_1}$$

$$= \frac{-(j0.15)^2}{j0.1696 + j0.1696 - 2(j0.1104) - j0.15} = j0.7120 \text{ per unit}$$

$$Z_{pp'}^{(0)} = \frac{-Z_0^2}{Z_{22}^{(0)} + Z_{33}^{(0)} - 2Z_{23}^{(0)} - Z_0}$$

$$= \frac{-(j0.50)^2}{j0.08 + j0.58 - 2(j0.08) - j0.50} = \infty$$

Thus, if the line from bus ② to bus ③ is opened, then an infinite impedance is seen looking into the zero-sequence network between points p and p' of the opening. Figure 12.6(b) confirms this fact since bus ③ would be isolated from the reference by opening the connection between bus ② and bus ③.

One open conductor

In this example Eq. (12.38) becomes

$$V_a^{(0)} = V_a^{(2)} = V_a^{(1)} = I_{23}\frac{Z_{pp'}^{(1)}Z_{pp'}^{(2)}}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)}}$$

$$= (0.4 - j0.3)\frac{(j0.7120)(j0.7120)}{j0.7120 + j0.7120}$$

$$= 0.1068 + j0.1424 \text{ per unit}$$

and from Eqs. (12.27) we now calculate the symmetrical components of the voltage at bus ③:

$$\Delta V_3^{(1)} = \Delta V_3^{(2)} = \frac{Z_{32}^{(1)} - Z_{33}^{(1)}}{Z_1}V_a^{(1)} = \left(\frac{j0.1104 - j0.1696}{j0.15}\right)(0.1068 + j0.1424)$$

$$= -0.0422 - j0.0562 \text{ per unit}$$

$$\Delta V_3^{(0)} = \frac{Z_{32}^{(0)} - Z_{33}^{(0)}}{Z_0}V_a^{(0)} = \left(\frac{j0.08 - j0.58}{j0.50}\right)(0.1068 + j0.1424)$$

$$= -0.1068 - j0.1424 \text{ per unit}$$

$$\Delta V_3 = \Delta V_3^{(0)} + \Delta V_3^{(1)} + \Delta V_3^{(2)} = -0.1068 - j0.1424 - 2(0.0422 + j0.0562)$$

$$= -0.1912 - j0.2548 \text{ per unit}$$

Since the prefault voltage at bus ③ equals $1.0 + j0.0$, the new voltage at bus ③ is

$$V_3' = V_3 + \Delta V_3 = (1.0 + j0.0) + (-0.1912 - j0.2548)$$

$$= 0.8088 - j0.2548 = 0.848\underline{/-17.5^\circ} \text{ per unit}$$

Two open conductors

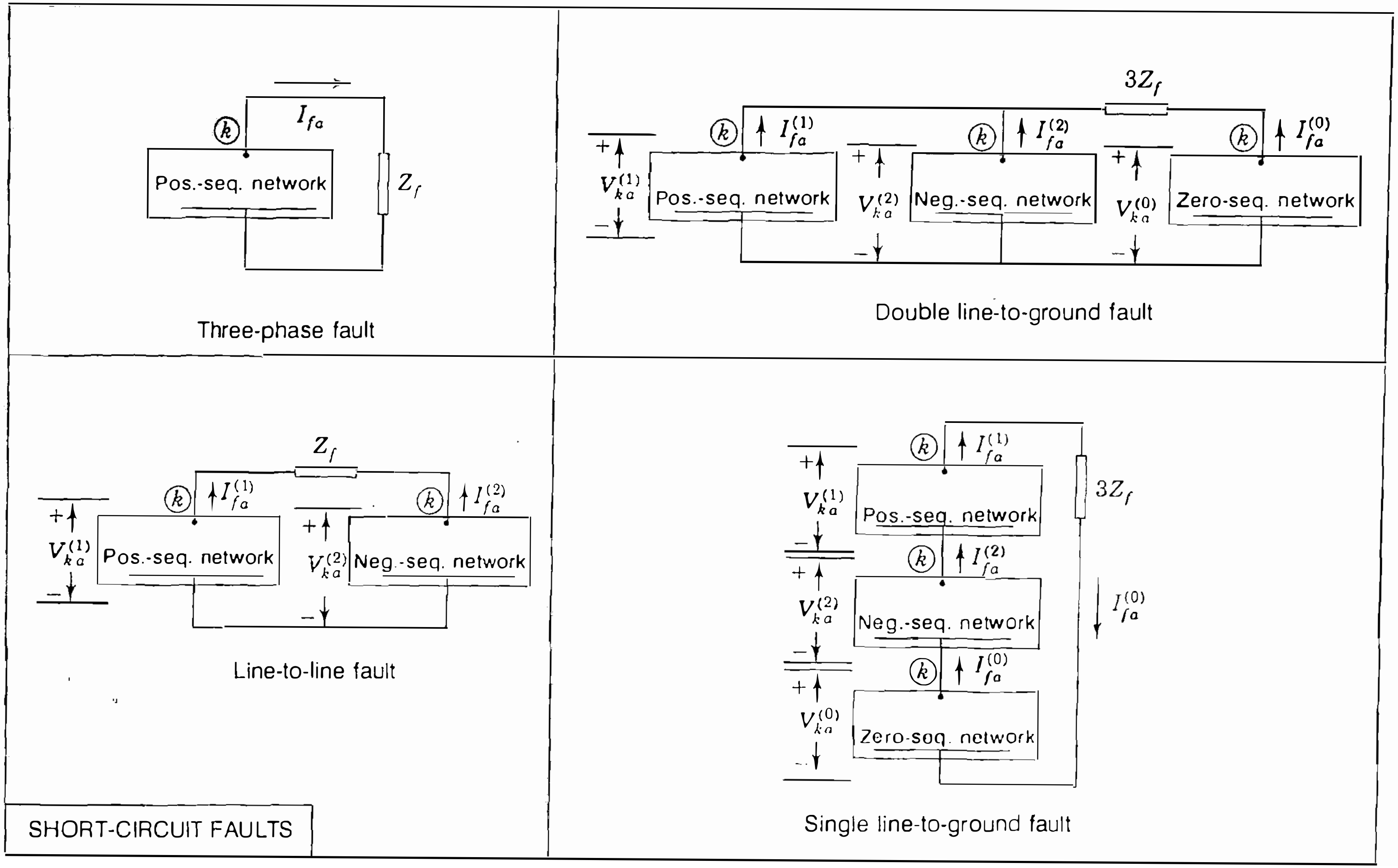
Inserting the infinite impedance of the zero-sequence network in *series* between points p and p' of the positive-sequence network causes an open circuit in the latter. No power transfer can occur in the system—confirmation of the fact that power cannot be transferred by only one phase conductor of the transmission line in this case since the zero-sequence network offers no return path for current.

12.7 SUMMARY

If the emfs in a positive-sequence network like that shown in Fig. 12.2 are replaced by short circuits, the impedance between the fault bus ⓚ and the reference node is the positive-sequence impedance $Z_{kk}^{(1)}$ in the equation developed for faults on a power system and is the series impedance of the Thévenin equivalent of the circuit between bus ⓚ and the reference node. Thus, we can regard $Z_{kk}^{(1)}$ as a single impedance or the entire positive-sequence network between bus ⓚ and the reference with no emfs present. If the voltage V_f is connected in series with this modified positive-sequence network, the resulting circuit, shown in Fig. 12.2(*e*), is the Thévenin equivalent of the original positive-sequence network. The circuits shown in Fig. 12.2 are equivalent only in their effect on any external connections made between bus ⓚ and the reference node of the original networks. We can easily see that no current flows in the branches of the equivalent circuit in the absence of an external connection, but current will flow in the branches of the *original* positive-sequence network if any difference exists in the phase or magnitude of the two emfs in the network. In Fig. 12.2(*b*) the current flowing in the branches in the absence of an external connection is the prefault load current.

When the other sequence networks are interconnected with the positive-sequence network of Fig. 12.2(*b*) or its equivalent shown in Fig. 12.2(*e*), the current flowing out of the network or its equivalent is $I_{fa}^{(1)}$ and the voltage between bus ⓚ and the reference is $V_{ka}^{(1)}$. With such an external connection, the current in any branch of the original positive-sequence network of Fig. 12.2(*b*) is the positive-sequence current in phase a of that branch during the fault. The prefault component of this current is included. The current in any branch of the Thévenin equivalent of Fig. 12.2(*e*), however, is only that portion of the actual positive-sequence current found by apportioning $I_{fa}^{(1)}$ of the fault among the branches represented by $Z_{kk}^{(1)}$ according to their impedances and does not include the prefault component.

In the preceding sections we have seen that the Thévenin equivalents of the sequence networks of a power system can be interconnected so that solving


Three-phase fault
Double line-to-ground fault
Line-to-line fault
Single line-to-ground fault
Pos.-seq. network
Neg.-seq. network
Zero-seq. network
I_{fa}
Z_f
$3Z_f$
$I_{fa}^{(1)}$
$I_{fa}^{(2)}$
$I_{fa}^{(0)}$
$V_{ka}^{(1)}$
$V_{ka}^{(2)}$
$V_{ka}^{(0)}$
k
SHORT-CIRCUIT FAULTS

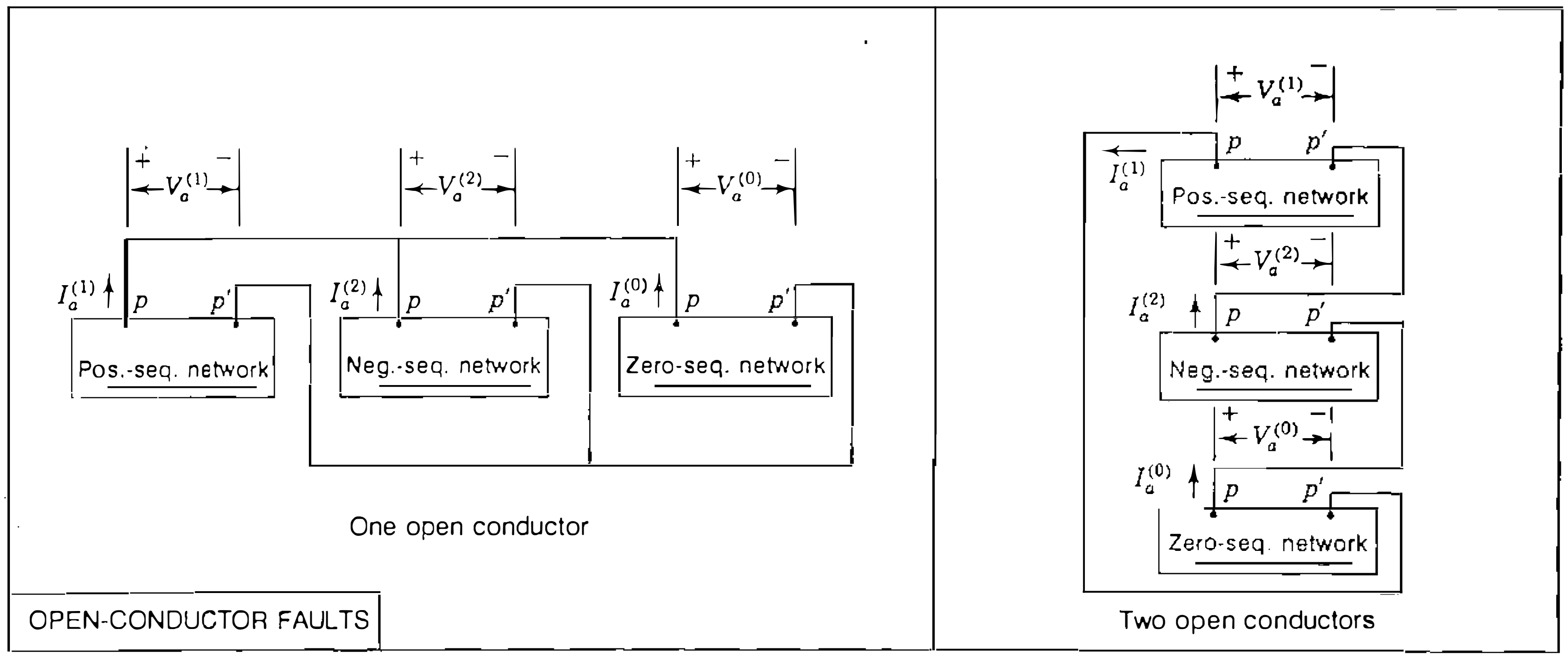


FIGURE 12.31
Summary of the connections of the sequence networks to simulate various types of short-circuit faults through impedance Z_f. $V_{ka}^{(0)}$, $V_{ka}^{(1)}$, and $V_{ka}^{(2)}$ are the symmetrical components of phase a voltage at the fault bus (k) with respect to reference. $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$ are the symmetrical components of the phase a voltage drops across the open-circuit points p and p'.

TABLE 12.1
Summary of equations for sequence voltages and currents at the fault point for various types of faults

	Short-circuit faults			Open-circuit faults	
	Line-to-ground fault	Line-to-line fault	Double line-to-ground fault	One open conductor	Two open conductors
Sequence currents	$I_{fa}^{(1)} = \dfrac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}$	$I_{fa}^{(1)} = \dfrac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_f}$	$I_{fa}^{(1)} = \dfrac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} \| \left(Z_{kk}^{(0)} + Z_f\right)}$	$I_a^{(1)} = \dfrac{I_{mn} Z_{pp'}^{(1)}}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)} \| Z_{pp'}^{(0)}}$	$I_a^{(1)} = \dfrac{I_{mn} Z_{pp'}^{(1)}}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$
	$I_{fa}^{(2)} = I_{fa}^{(1)}$	$I_{fa}^{(2)} = -I_{fa}^{(1)}$	$I_{fa}^{(2)} = -I_{fa}^{(1)} \dfrac{\left(Z_{kk}^{(0)} + 3Z_f\right)}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}$	$I_a^{(2)} = -I_a^{(1)} \dfrac{Z_{pp'}^{(0)}}{Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$	$I_a^{(2)} = I_a^{(1)}$
	$I_{fa}^{(0)} = I_{fa}^{(1)}$	$I_{fa}^{(0)} = 0$	$I_{fa}^{(0)} = -I_{fa}^{(1)} \dfrac{Z_{kk}^{(2)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}$	$I_a^{(0)} = -I_a^{(1)} \dfrac{Z_{pp'}^{(2)}}{Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$	$I_a^{(0)} = I_a^{(1)}$
Sequence voltages	$V_{ka}^{(1)} = I_{fa}^{(1)}(Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f)$	$V_{ka}^{(1)} = I_{fa}^{(1)}(Z_{kk}^{(2)} + Z_f)$	$V_{ka}^{(1)} = V_{ka}^{(0)} - 3I_{fa}^{(0)} Z_f$	$V_a^{(1)} = I_a^{(1)} \dfrac{Z_{pp'}^{(2)} Z_{pp'}^{(0)}}{Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$	$V_a^{(1)} = I_a^{(1)}(Z_{pp'}^{(2)} + Z_{pp'}^{(0)})$
	$V_{ka}^{(2)} = -I_{fa}^{(1)} Z_{kk}^{(2)}$	$V_{ka}^{(2)} = I_{fa}^{(1)} Z_{kk}^{(2)}$	$V_{ka}^{(2)} = -I_{fa}^{(2)} Z_{kk}^{(2)}$	$V_a^{(2)} = -I_a^{(2)} Z_{pp'}^{(2)}$	$V_a^{(2)} = -I_a^{(2)} Z_{pp'}^{(2)}$
	$V_{ka}^{(0)} = -I_{fa}^{(1)} Z_{kk}^{(0)}$	$V_{ka}^{(0)} = 0$	$V_{ka}^{(0)} = -I_{fa}^{(0)} Z_{kk}^{(0)}$	$V_a^{(0)} = -I_a^{(0)} Z_{pp'}^{(0)}$	$V_a^{(0)} = -I_a^{(0)} Z_{pp'}^{(0)}$

Note: "$\|$" implies parallel combination of impedances.
$V_{ka}^{(0)}$, $V_{ka}^{(1)}$, and $V_{ka}^{(2)}$ are the symmetrical components of phase *a* voltage at the fault bus (k) with respect to the reference.
$V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$ are the symmetrical components of the phase *a* voltage drops across the open-circuit points *p* and *p'*.

the resulting network yields the symmetrical components of current and voltage *at the fault*. The connections of the sequence networks to simúlate various types of short-circuit faults, including a symmetrical three-phase fault, are shown in Fig. 12.31. The sequence networks are indicated schematically by rectangles enclosing a heavy line to represent the reference of the network and a point marked bus ⓚ to represent the point in the network where the fault occurs. The positive-sequence network contains emfs that represent the internal voltages of the machines.

Regardless of the prefault voltage profile or the particular type of short-circuit fault occurring, the only current which causes positive-sequence voltage *changes* at the buses of the system is the symmetrical component $I_{fa}^{(1)}$ of the current I_{fa} coming out of phase a of the system at the fault bus ⓚ. These positive-sequence voltage changes can be calculated simply by multiplying column k of the positive-sequence bus impedance matrix $\mathbf{Z}_{\text{bus}}^{(1)}$ by the *injected* current $-I_{fa}^{(1)}$. Similarly, the negative- and zero-sequence components of the voltage changes due to the short-circuit fault on the system are caused, respectively, by the symmetrical components $I_{fa}^{(2)}$ and $I_{fa}^{(0)}$ of the fault current I_{fa} *out* of bus ⓚ. These sequence voltage changes are also calculated by multiplying columns ⓚ of $\mathbf{Z}_{\text{bus}}^{(2)}$ and $\mathbf{Z}_{\text{bus}}^{(0)}$ by the respective current injections $-I_{fa}^{(2)}$ and $-I_{fa}^{(0)}$.

In a very real sense, therefore, there is only one procedure for calculating the symmetrical components of the voltage changes at the buses of the system when a short-circuit fault occurs at bus ⓚ—that is, to find $I_{fa}^{(0)}$, $I_{fa}^{(1)}$, and $I_{fa}^{(2)}$ and to multiply columns ⓚ of the corresponding bus impedance matrices by the negative values of these currents. For the common types of short-circuit faults the only differences in the calculations concern the method of simulating the fault at bus ⓚ and of formulating the equations for $I_{fa}^{(0)}$, $I_{fa}^{(1)}$, and $I_{fa}^{(2)}$. The connections of the Thévenin equivalents of the sequence networks, which provide a ready means of deriving the equations for $I_{fa}^{(0)}$, $I_{fa}^{(1)}$, and $I_{fa}^{(2)}$, are summarized in Fig. 12.31 and the equations themselves are set forth in Table 12.1.

Faults due to open conductors involve *two* injections into each sequence network at the buses nearest the opening in the conductor. Otherwise, the procedure for calculating sequence voltage changes in the system is the same as that for short-circuit faults. Equations for the sequence voltages and currents at the fault are also summarized in Table 12.1.

The reader is reminded of the need to adjust the phase angles of the symmetrical components of the currents and the voltages in those parts of the system which are separated from the fault bus by Δ-Y transformers.

PROBLEMS

12.1. A 60-Hz turbogenerator is rated 500 MVA, 22 kV. It is Y-connected and solidly grounded and is operating at rated voltage at no load. It is disconnected from the rest of the system. Its reactances are $X_d'' = X_1 = X_2 = 0.15$ and $X_0 = 0.05$ per unit. Find the ratio of the subtransient line current for a single line-to-ground fault to the subtransient line current for a symmetrical three-phase fault.

12.2. Find the ratio of the subtransient line current for a line-to-line fault to the subtransient current for a symmetrical three-phase fault on the generator of Prob. 12.1.

12.3. Determine the inductive reactance in ohms to be inserted in the neutral connection of the generator of Prob. 12.1 to limit the subtransient line current for a single line-to-ground fault to that for a three-phase fault.

12.4. With the inductive reactance found in Prob. 12.3 inserted in the neutral of the generator of Prob. 12.1, find the ratios of the subtransient line currents for the following faults to the subtransient line current for a three-phase fault: (*a*) single line-to-ground fault, (*b*) line-to-line fault, and (*c*) double line-to-ground fault.

12.5. How many ohms of resistance in the neutral connection of the generator of Prob. 12.1 would limit the subtransient line current for a single line-to-ground fault to that for a three-phase fault?

12.6. A generator rated 100 MVA, 20 kV has $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 5\%$. Its neutral is grounded through a reactor of 0.32 Ω. The generator is operating at rated voltage without load and is disconnected from the system when a single line-to-ground fault occurs at its terminals. Find the subtransient current in the faulted phase.

12.7. A 100-MVA 18-kV turbogenerator having $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 5\%$ is about to be connected to a power system. The generator has a current-limiting reactor of 0.162 Ω in the neutral. Before the generator is connected to the system, its voltage is adjusted to 16 kV when a double line-to-ground fault develops at terminals *b* and *c*. Find the initial symmetrical root-mean-square (rms) current in the ground and in line *b*.

12.8. The reactances of a generator rated 100 MVA, 20 kV are $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 5\%$. The generator is connected to a Δ-Y transformer rated 100 MVA, 20Δ-230Y kV, with a reactance of 10%. The neutral of the transformer is solidly grounded. The terminal voltage of the generator is 20 kV when a single line-to-ground fault occurs on the open-circuited, high-voltage side of the transformer. Find the initial symmetrical rms current in all phases of the generator.

12.9. A generator supplies a motor through a Y-Δ transformer. The generator is connected to the Y side of the transformer. A fault occurs between the motor terminals and the transformer. The symmetrical components of the subtransient current in the motor flowing toward the fault are

$$I_a^{(1)} = -0.8 - j2.6 \text{ per unit}$$

$$I_a^{(2)} = -j2.0 \text{ per unit}$$

$$I_a^{(0)} = -j3.0 \text{ per unit}$$

From the transformer toward the fault

$$I_a^{(1)} = 0.8 - j0.4 \text{ per unit}$$

$$I_a^{(2)} = -j1.0 \text{ per unit}$$

$$I_a^{(0)} = 0 \text{ per unit}$$

Assume $X_d'' = X_1 = X_2$ for both the motor and the generator. Describe the type of fault. Find (*a*) the prefault current, if any, in line *a*; (*b*) the subtransient fault current in per unit; and (*c*) the subtransient current in each phase of the generator in per unit.

12.10. Using Fig. 12.18, calculate the bus impedance matrices $\mathbf{Z}_{\text{bus}}^{(1)}$, $\mathbf{Z}_{\text{bus}}^{(2)}$, and $\mathbf{Z}_{\text{bus}}^{(0)}$ for the network of Example 12.6.

12.11. Solve for the subtransient current in a single line-to-ground fault first on bus ① and then on bus ② of the network of Example 12.6. Use the bus impedance matrices of Prob. 12.10. Also, find the voltages to neutral at bus ② with the fault at bus ①.

12.12. Calculate the subtransient currents in all parts of the system of Example 12.6 with prefault current neglected if the fault on the low-voltage side of the transformer is a line-to-line fault. Use $\mathbf{Z}_{\text{bus}}^{(1)}$, $\mathbf{Z}_{\text{bus}}^{(2)}$, and $\mathbf{Z}_{\text{bus}}^{(0)}$ of Prob. 12.10.

12.13. Repeat Prob. 12.12 for a double line-to-ground fault.

12.14. Each of the machines connected to the two high-voltage buses shown in the single-line diagram of Fig. 12.32 is rated 100 MVA, 20 kV with reactances of $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 4\%$. Each three-phase transformer is rated 100 MVA, 345Y/20Δ kV, with leakage reactance of 8%. On a base of 100 MVA, 345 kV the reactances of the transmission line are $X_1 = X_2 = 15\%$ and $X_0 = 50\%$. Find the 2 × 2 bus impedance matrix for each of the three sequence networks. If no prefault current is flowing in the network, find the subtransient current to ground for a double line-to-ground fault on lines *B* and *C* at bus ①. Repeat for a fault at bus ②. When the fault is at bus ②, determine the current in phase *b* of machine 2 if the lines are named so that $V_A^{(1)}$ leads $V_a^{(1)}$ by 30°. If the phases are named so that $I_a^{(1)}$ leads $I_A^{(1)}$ by 30°, what letter (*a*, *b*, or *c*) would identify the phase of machine 2 which would carry the current found for phase *b* above?

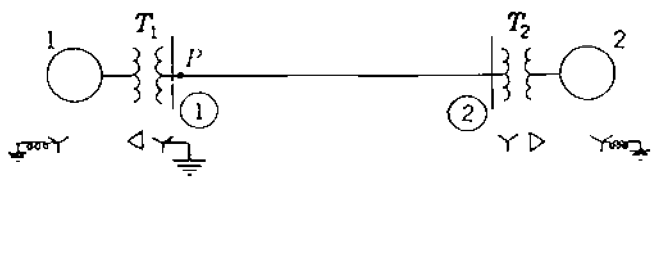


FIGURE 12.32
Single-line diagram for Prob. 12.14.

12.15. Two generators G_1 and G_2 are connected, respectively, through transformers T_1 and T_2 to a high-voltage bus which supplies a transmission line. The line is open at the far end at which point *F* a fault occurs. The prefault voltage at point *F* is